

# Capital-Labor Substitution, Structural Change and the Labor Income Share\*

Francisco Alvarez-Cuadrado, Ngo Van Long and Markus Poschke

*Department of Economics, McGill University, Montreal H3A 2T7, Canada*

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## Abstract

Recent work has documented declines in the labor income share in the United States and beyond. This paper documents that this decline was more pronounced in manufacturing than in services in the U.S. and in a broad set of other industrialized economies, and shows that a model with cross-sectoral differences in productivity growth and in the degree of capital-labor substitutability is consistent with these trends. We calibrate the model exploiting additional information on the pace of structural change from manufacturing to services, on which the model also has predictions. We then conduct a decomposition to establish the relative importance of several potential drivers of changes in factor income shares and structural change that have been proposed in the literature. This exercise reveals that differences in the degree of capital bias of technical change across sectors, combined with differences in substitution possibilities, are key determinants of the observed patterns.

JEL Classification: O40, O41, O30

Keywords: Labor income share, biased technical change, capital-labor substitution, structural change

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# 1 Introduction

In two recent contributions, Elsby, Hobijn and Şahin (2013) and Karabarbounis and Neiman (2014) have documented the decline in the labor income share in the United States and in other countries. In this paper, we document that this decline was much more pronounced in manufacturing than in services, and propose an explanation that is consistent with these sectoral differences and with observed structural change from manufacturing to services over the period 1960 to 2005. The key element for explaining the observed differential evolution of sectoral labor income shares are differences in the degree of capital bias of technical change across sectors, combined with sectoral differences in the substitutability of capital and labor.

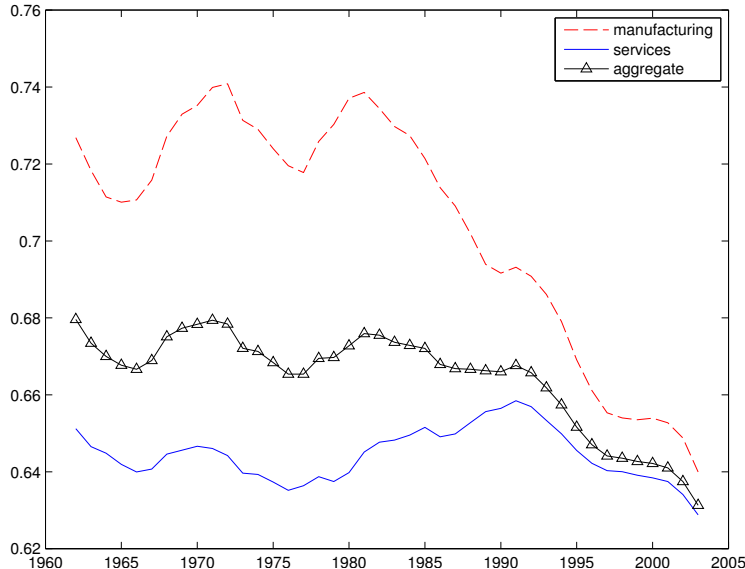
While aggregate factor income shares have long been thought to be constant, recent work has documented that in the last few decades, the labor income share has declined substantially. This is illustrated in Figure 1 using U.S. data from Jorgenson (2007). The trend shown here is in line with the highly detailed analysis by Elsby et al. (2013) for the U.S., and with the findings by Karabarbounis and Neiman (2014) for a broad set of countries. Figure 1 also shows that the aggregate pattern is driven by differential developments at the sectoral level: Jorgenson's KLEM data reveal that while in 1960, the labor share of income in manufacturing exceeded the aggregate labor income share, whereas the reverse was true for services, this pattern has changed substantially over the last 45 years. In this time, the labor share of income in manufacturing has declined substantially, and the one in services slightly. The decline has been pronounced; the labor income share in manufacturing has declined on average by 2.1 percentage points per decade, and the aggregate one by 1.2 percentage points. Section 2 describes the data underlying this pattern in more detail and shows that correcting income shares following Valentinyi and Herrendorf (2008) leaves these broad patterns intact. More than that, we show using EUKLEMS data that the labor income share has declined in all but three of a set of 16 industrialized economies, with the labor income share in manufacturing declining by even more in most of them.<sup>1</sup>

Unfortunately, most popular multi-sector models – for instance those used for the analysis of structural change in Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) – are unable to account for such changes in sectoral income shares. The key reason for this is that they assume that sectoral production functions are

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<sup>1</sup>Karabarbounis and Neiman (2014) have shown some similar results. However, they provide less country-level detail and do not contrast the evolution of the labor income share in manufacturing with that in services. Similarly, using BEA industry accounts, Elsby et al. (2013) show that the labor income share in U.S. manufacturing declined severely in the period 1987-2011. In earlier work, Blanchard (1997) and Caballero and Hammour (1998) have documented medium term variation in the labor income share in some continental European economies and linked them to labor market rigidities.

Figure 1: The labor income share in the U.S.



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

Cobb-Douglas.<sup>2</sup> In this case, as is well known, sectoral labor income shares are forced to be constant if factor markets are competitive. If the elasticity of output with respect to capital differs across sectors, the aggregate labor income share may still change with structural change. However, as we show below, this channel can only account for a small fraction of the observed change in the aggregate labor income share, although structural change in the United States since 1960 was substantial.<sup>3</sup> These theories thus have no chance of replicating the observed changes in sectoral and aggregate labor income shares.

To address this issue, we propose a theory where the elasticity of substitution between capital and labor is different from one (i.e. production functions are not Cobb-Douglas), and potentially differs between manufacturing and services. Assuming that the elasticity of substitution within each sector is constant, this implies that sector  $i$  output is produced using the production function

$$Y_{it} = \left[ \alpha_i (B_{it} K_{it})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) (A_{it} L_{it})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}} \equiv A_{it} \left[ \alpha_i (\tilde{B}_{it} K_{it})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) L_{it}^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad (1)$$

<sup>2</sup>Kongsamut et al. (2001) assume instead that sectoral production functions are proportional. This is similarly restrictive, as it does not allow for sectoral differences in the level or the evolution of factor income shares.

<sup>3</sup>The value added share of manufacturing declined by 25 percentage points, or about half, over the sample period. See Section 2 for more background on structural change in the United States.

where  $\alpha_i$  governs the relative importance of the two inputs,  $K_{it}$  and  $L_{it}$  are capital and labor used in sector  $i$  at time  $t$ , and  $\sigma_i \in (0, \infty)$  is the elasticity of substitution between these two inputs.<sup>4</sup> Both formulations of the CES technology are equivalent. In the first,  $A_{it}$  and  $B_{it}$  stand for the the levels of labor- and capital-augmenting productivity, respectively. In the second specification, which we will use throughout the paper,  $A_{it}$  is the level of Hicks-neutral productivity, and we refer to the growth rate of  $\tilde{B}_{it} \equiv B_{it}/A_{it}$  as the factor imbalance of technical change. Finally, the bias of technical change is given by  $\frac{1-\sigma_i}{\sigma_i}g(\tilde{B}_{it})$ , where we use  $g(\cdot)$  to denote a growth rate. If the bias is positive (negative), technical change is labor-biased (capital-biased) in the sense that it increases (decreases) the marginal product of labor relative to that of capital.<sup>5</sup>

With this production function, the ratio of factor income shares in sector  $i$  is given by

$$\frac{KIS_{it}}{LIS_{it}} = \frac{R_t K_{it}}{w_t L_{it}} = \frac{\alpha_i}{1 - \alpha_i} \left( \frac{B_{it}}{A_{it}} \frac{K_{it}}{L_{it}} \right)^{\frac{\sigma_i - 1}{\sigma_i}} = \frac{\alpha_i}{1 - \alpha_i} \left( \tilde{B}_{it} \frac{K_{it}}{L_{it}} \right)^{\frac{\sigma_i - 1}{\sigma_i}} \quad (2)$$

if factor markets are competitive and firms choose inputs optimally. Here,  $KIS_{it}$  and  $LIS_{it}$  stand for the capital and labor income shares in sector  $i$  at time  $t$ , and  $R_t$  and  $w_t$  for the rental rate and the wage rate, respectively. This expression is potentially consistent with observed trends: if capital and labor are gross complements, i.e.  $\sigma_i < 1$ , as most empirical evidence suggests (see below for a detailed discussion), then the labor income share in sector  $i$  decreases as long as technical change in that sector is sufficiently capital-biased relative to the rate of capital accumulation ( $g(\tilde{B}_i) < g(K_{it}/L_{it})$ ). If this is the case, the marginal product of labor relative to that of capital increases less than capital-labor ratio, implying a declining labor income share.<sup>6</sup> An equivalent intuition is that a decrease in effective capital per unit of effective labor ( $g(B_{it}K_{it}) < g(A_{it}L_{it})$ ) induces a more than proportional increase

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<sup>4</sup>This elasticity was first introduced by Hicks in *The Theory of Wages* (1932) in an attempt to explore the functional distribution of income in a growing economy. It is defined as the elasticity of the capital-labor ratio in sector  $i$ ,  $k_i$ , to the ratio of factor prices, the wage to rental rate ratio,  $\omega \equiv \frac{w}{R}$ :

$$\sigma_i \equiv \frac{\partial k_i}{\partial \omega} \frac{\omega}{k_i}$$

Here, we assume that  $\omega$  is common across sectors.

<sup>5</sup>See Acemoglu (2002) for a detailed discussion of the relationship between factor-augmenting and factor-biased technical change. In short, under competitive factor markets, optimizing firms equate the marginal rate of technical substitution to the ratio of factor prices:  $\frac{1-\alpha_i}{\alpha_i} \tilde{B}_{it}^{\frac{1-\sigma_i}{\sigma_i}} (K_{it}/L_{it})^{\frac{1}{\sigma_i}} = \omega$ . In growth rates,  $g(\omega) = \frac{1}{\sigma_i}g(K_{it}/L_{it}) + \frac{1-\sigma_i}{\sigma_i}g(\tilde{B}_{it})$ . Hicks-neutral productivity  $A_{it}$  does not enter this expression directly – it is neutral in the sense that its change leaves the the marginal rate of technical substitution unchanged. This is in contrast to changes in the bias of technical change,  $\frac{1-\sigma_i}{\sigma_i}g(\tilde{B}_{it})$ .

<sup>6</sup>A falling labor income share is also consistent with  $\sigma_i > 1$  and rising  $\tilde{B}_{it}K_{it}/L_{it}$ , as in Karabarbounis and Neiman (2014), and as argued for by Piketty (2014). However, this scenario runs counter to the fact that virtually all estimates of aggregate  $\sigma$  in the recent literature lie below 1 (see Table 1 in León-Ledesma,

in the ratio of the rental rate to the wage, and therefore a reduction in the sectoral labor income share.<sup>7</sup>

The evolution of sectoral labor income shares in this setting thus depends on the elasticity of substitution between capital and labor in the sector,  $\sigma_i$ , on the bias of technical change in the sector, and on capital accumulation in the sector. While the former two drivers are primitives in macroeconomic models with exogenous growth, the latter is determined endogenously, by two margins: the accumulation of capital in the economy as a whole, and the allocation of capital and labor across sectors. We therefore embed the sectoral production structure in (1) in a dynamic general equilibrium model with two sectors, and quantitatively analyze the dynamics of this model. Due to the potential impact of structural change on the aggregate labor income share, we also allow for the most prominent additional drivers of structural change proposed in the literature, i.e. non-homothetic preferences and cross-sectoral differences in capital-intensity in addition to differences in productivity growth and in factor substitutability. We then calibrate the model to the experience of changing labor income shares and structural change of the United States over the period 1960 to 2005 to draw lessons on the determinants of changing factor income shares over this period.

This quantitative exercise yields rich results. First, capital and labor are strong complements in both sectors,  $\sigma_i < 1$ . Second, the calibration exercise points to a strong capital-bias of productivity growth ( $\frac{1-\sigma_i}{\sigma_i}g(\tilde{B}_i) < 0$ ) in both sectors as the main driver of the decline in sectoral labor income shares. Third, the larger decline in the labor income share in manufacturing relative to that in services is driven to a similar extent by a larger factor imbalance in productivity growth (lower  $g(\tilde{B}_i)$ ) and by a larger degree of flexibility (larger  $\sigma_i$ ) in manufacturing. Hence, differences across sectors in both of these features are key for understanding the evolution of sectoral labor income shares.<sup>8</sup> Fourth, differences in productivity growth across sectors clearly are the main determinant of structural change. Finally, non-homotheticities in preferences and differences in capital intensity hardly affect the evolution of factor income shares. Their effect on structural change is similarly small.<sup>9</sup>

Overall, the quantitative analysis points to the combination of the complementarity be-

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McAdam and Willman (2010) and the review of some more recent contributions at the end of this section). See also footnote 29.

<sup>7</sup>A similar mechanism is at work in the literature on capital-skill complementarity, for instance Krusell, Ohanian, Ríos-Rull and Violante (2000). In that context, the gross complementarity (substitutability) between equipment and skilled (unskilled) labor leads to an increase in the skill premium in response to capital accumulation.

<sup>8</sup>While according to equation (2), a difference in either one of these two factors is sufficient for generating differential changes in factor income shares, the factor allocations generated by the model in this case are not consistent with the data.

<sup>9</sup>This may well be different for structural change out of agriculture in an earlier period.

tween capital and labor in production with capital-biased technical change as the main driver of declining sectoral labor income shares. Both of these ingredients are in line with the existing literature. A value of  $\sigma$  below 1 is found by the bulk of the empirical literature (see below for details). Our finding of capital-biased technical change is consistent with the predictions of models of directed technical change (see in particular Acemoglu 2002). These models predict that when capital and labor are complements in production and the capital-labor ratio increases, the activity of profit-maximizing innovators results in capital-biased technical change.

It appears that an alternative to our analysis would have been to use (2) or the first order conditions for firms' choice of capital and labor from which it is derived to estimate  $\sigma_i$  and sectoral productivity growth rates, and to use parameter estimates to draw inference on the evolution of factor income shares. Our approach has two important advantages compared to this alternative. The first advantage is related to the identification of model parameters, in particular the elasticity of substitution and the bias of technical change. Our calibration exercise uses not only both first order conditions, but also exploits two further sources of information. The first consists in also using the the production function, just as recommended by Klump, McAdam and Willman (2007) and León-Ledesma et al. (2010) for the estimation of production systems. These authors argue, and illustrate with extensive Monte Carlo simulations, that single equation approaches are largely unsuitable for jointly uncovering the elasticity of substitution and the bias of technical change. They stress the superiority of a multi-equation approach in terms of robustly capturing production and technical parameters. The second additional source of information comes from the general equilibrium nature of our exercise. This implies that observations on structural change contribute to identifying the production function parameters. This is crucial for separately identifying the impact of sectoral differences in the bias of productivity growth and differences in factor substitutability on changes in factor income shares.

The second advantage is conceptual and arises in the decomposition exercises. In addition to ignoring links across sectors, the alternative type of analysis proposed above would ignore the endogeneity of the evolution of the capital-labor ratio, and therefore would tend to attribute insufficient importance to productivity dynamics (which are the fundamental determinant of capital accumulation and allocation) as a driver of labor income shares. This is akin to a growth accounting exercise that ignores that capital accumulation is ultimately driven by productivity growth.<sup>10</sup>

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<sup>10</sup>These advantages do not come entirely without costs. In a framework that incorporates several, but not all possible sources of structural change, substantial differences in technical change across sectors are required

**Related literature.** It is convenient to first discuss a number of recent, closely related papers estimating the elasticity of substitution between capital and labor, in particular Herrendorf, Herrington and Valentinyi (2015), Oberfield and Raval (2014) and León-Ledesma, McAdam and Willman (2015). All of these authors focus on the estimation, and find similar substitution elasticities to the ones that result from our calibration exercise. None of them conducts a counterfactual decomposition exercise aiming to identify fundamental driving forces of changes in sectoral labor income shares. León-Ledesma et al. (2015) estimate an aggregate elasticity of substitution around 0.7 for the U.S. economy applying the technique they advocate in León-Ledesma et al. (2010) to aggregate U.S. time series. Herrendorf et al. (2015) estimate CES production functions for the aggregate U.S. economy and for the sectors agriculture, manufacturing and services. Differently from our approach, they do so by sector, not jointly, and do not spell out the implications for the evolution of aggregate or sectoral factor income shares. They do however share our conclusion that productivity growth differences across sectors are the key driver of structural change. Finally, Oberfield and Raval (2014) estimate the elasticity of substitution in U.S. manufacturing by applying a novel identification strategy to plant-level data. They find a value of 0.7, close to the number in our calibration exercise. While this is not the focus of their paper, they also show that, given this estimate, observed changes in factor prices (an increasing wage to rental ratio) predict an increasing labor income share. This is of course generally the case when  $\sigma_i < 1$ . As a consequence, these authors attribute most of the observed decline in the labor income share to a residual catch-all “bias” term within manufacturing industries. This conclusion is consistent with our finding regarding the relevance of capital-biased technical change growth as an important driver of changes in the labor income share.<sup>11</sup>

The estimates in these papers are representative of those in the literature on the estimation of  $\sigma$ . For instance, León-Ledesma et al. (2010, Table 1) and the survey Raval (2015) generally find estimates between 0.5 and 0.8, with very few estimates above 1.

In the emerging literature on the declining labor income share, a few papers are close to ours in the sense that they focus on the role of capital accumulation and technical change.

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for the model to be consistent with the extent of sectoral reallocation in the data. As a consequence, the calibration implies that the overall rate of productivity growth in services is negative. Although it has been documented that certain service industries, like utilities, have experienced considerable negative productivity growth, it is difficult to believe that this is true for the sector as a whole. This downside would conceivably be absent in a model that incorporated even more drivers of structural change.

<sup>11</sup>Chirinko and Mallick (2017) estimate an aggregate elasticity of substitution of 0.4. Choi and Ríos-Rull (2009) document the evolution of the labor income share over the business cycle and evaluate the ability of a variety of real business cycle models to account for its response to productivity shocks. As in our case, their preferred specification includes a CES technology where capital and labor are gross complements.

Lawrence (2015) and, to some extent, Harrison (2002) suggest that the decline in the labor income share is consistent with some combination of technical change and low capital-labor substitutability. Our model formalizes and quantifies these ideas. Karabarbounis and Neiman (2014) attribute the declining labor income share to the secular decline in the price of investment goods. This conclusion relies on a capital/labor elasticity of substitution above one, which they identify from cross-country comovements of labor income shares and rental rates. When capital and labor are gross substitutes, the secular decline in the price of investment goods –investment specific technical change– that drives their rental rate, is, in our language, capital-biased technical change. In comparison, we find it comforting that our alternative explanation relies on low substitutability between capital and labor, in line with the vast majority of estimates in the literature.

Another set of papers considers other types of explanations. Elsby et al. (2013) carefully document the decline in the labor income share in the U.S. and point to off-shoring as potentially important, among several potential explanations they explore using cross-industry data.<sup>12</sup> Similarly, Harrison (2002) links international trade to changes in the labor income share. Autor, Dorn, Katz, Patterson and Van Reenen (2017) and Barkai (2016) focus on the importance of increases in industry concentration and the resulting changes in market power. In principle, declines in the wage to rental rate stemming from this channel could be inaccurately ascribed to biased technical change in the calibration of a frictionless model. Yet, the fact that Autor et al. (2017) find a very limited effect of changes in concentration on the labor income share mitigates this concern.

Finally, since our paper also provides a quantitative analysis of structural change, it is also related to recent work on that topic, in particular Buera and Kaboski (2009, 2012*a*, 2012*c*), and Swiecki (2017). Buera and Kaboski (2009) conduct a first quantitative evaluation of two potential drivers of structural change: productivity growth differences and non-homothetic preferences. Buera and Kaboski (2012*a*, 2012*c*) also analyze the “Rise of the Service Economy,” but focus on skill differences across different segments of the service sector and on differences in scale across sectors, respectively. Swiecki (2017) quantitatively analyzes a set of four drivers of structural change that partly overlaps with those we consider. In line with our results, he stresses the importance of differences in sectoral growth rates for structural change. In addition, he concludes that non-homotheticities matter mostly for structural change out of agriculture. Trade, a factor that we abstract from, matters only for some individual countries. Overall, our work appears to be the first contribution linking the evolution

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<sup>12</sup>The fact that labor income shares have declined across countries limits how broadly applicable the offshoring channel can be.



of factor income shares and structural change.

In the next section, we provide more detail on the evolution of sectoral factor income shares. In Section 3, we describe our model. In Section 4, we evaluate the power of the mechanism quantitatively. Finally, Section 5 concludes, while the appendices contain additional derivations and information on data sources.

## 2 The evolution of factor income shares in manufacturing and services

Recent literature has documented in detail the decline in the aggregate labor income share in the United States (Elsby et al. 2013) and across countries (Karabarbounis and Neiman 2014). In this section, we document that this decline was to a very large extent driven by a decline of sectoral labor income shares, in particular of that in manufacturing. We start by showing this for the U.S., and then add evidence for a cross-section of 16 other developed countries.<sup>13</sup> The section concludes with some background on structural change in the U.S..

### 2.1 Sectoral labor income shares in the United States

Figure 1 showed the evolution of the labor income share in manufacturing and services in the United States from 1960 to 2005. Over this period, the aggregate labor income share declined by 1.2 percentage points per decade, the one in manufacturing by 2.1 percentage points per decade, and that in services by 0.5.

The measure of the labor income share shown in Figure 1 is computed as total labor compensation in a sector divided by the sum of the value of capital services and labor compensation in that sector. We call this measure “naive” because it ignores links across industries. For example, food manufacturing uses inputs from the transportation industry which in turn are produced using capital, labor and other intermediate inputs. Therefore, the true labor income share in the production of manufacturing value added also depends on “naive” labor income shares in sectors producing intermediate inputs used in manufacturing. We thus also compute a measure that takes these links into account, following Valentinyi and Herrendorf (2008) (see the Appendix A.1 for details).

To do so, we use data from Jorgenson’s (2007) 35-sector KLEM database. These data are based on a combination of industry data from the BEA and the BLS and are described

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<sup>13</sup>Elsby et al. (2013) draw a similar conclusion for the U.S. since 1987. Karabarbounis and Neiman (2014) also show cross-industry developments (their Figure 5), but provide less country-level detail and no value for a broad service sector.

in detail in Jorgenson, Gollop and Fraumeni (1987), Jorgenson (1990) and Jorgenson and Stiroh (2000). They cover 35 sectors at roughly the 2-digit SIC level from 1960 to 2005. The raw data are accessible at <http://hdl.handle.net/1902.1/10684>. The data contain, for each industry and year, labor compensation, the value of capital services, and the value of intermediate inputs by source industry. These add up to the value of gross output in that industry. Data on capital and labor are carefully adjusted for input quality differences across sectors, allowing us to treat them as homogenous inputs in the following. Knowing the input-output structure allows computing the labor income share in production of sectoral value added.<sup>14</sup>

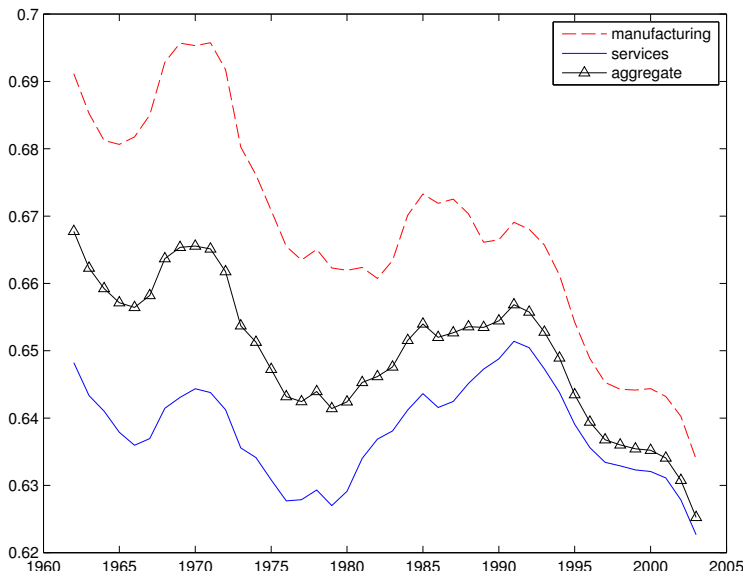
Results for the labor income share in value added are shown in Figure 2. Results using this measure are different in details, but the overall patterns remain unchanged. The main difference is that by this measure, the labor income share in manufacturing, while still exceeding that in services, is not as high as by the naive measure, since it takes into account that manufacturing value added also uses inputs from other sectors, with lower naive labor income shares. By this measure, the labor income share in manufacturing has fallen by 1.4 percentage points per decade, the one in services by 0.6 percentage points per decade, and the aggregate one by 1 percentage points per decade. The last two values are very close to the counterparts for the naive labor income share. Only the manufacturing number changes somewhat. However, the qualitative pattern is clearly maintained, with a decline in the aggregate labor income share driven largely by a decline in the labor income share in manufacturing.

Could the change in the aggregate labor income share be purely due to structural change? After all, the labor income share in manufacturing is higher than that in services, so that structural change from manufacturing to services will reduce the aggregate labor income share. A simple calculation shows that this channel is quantitatively minor. The aggregate labor income share is a value-added weighted average of the sectoral labor income shares. The share of manufacturing in value added has declined by almost 25 percentage points over the sample period (see Figure 4). Given a gap of 4.3 percentage points between the initial labor income shares in manufacturing and in services, this implies that structural change could account for a change in the aggregate labor income share of 1.075 percentage points over the period 1960 to 2005. This amount corresponds approximately to the average change in the labor income share over a single decade. Thus, structural change on its own

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<sup>14</sup>For the definitions of our sectors manufacturing and services, see Appendix A.2. Numbers reported here for the aggregate economy refer to the aggregate of manufacturing and services. Results are not sensitive to details of sector definition like the treatment of utilities, government or mining.

Figure 2: Non-naive labor income share, by sector, 1960-2005



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

can account for less than a quarter of the observed change in the aggregate labor income share.

## 2.2 Other countries

Changes in the labor income share have not been limited to the United States. In this section, we provide evidence from 16 other developed economies. We do so using EU KLEMS data. The March 2011 data release, available at <http://www.euklems.net>, contains data up to 2007 for 72 industries. We again aggregate those up to manufacturing and services. While this data contains a lot of industry detail, it does not contain an input/output structure, so we are limited to computing the naive labor income share, computed as compensation of employees over value added. However, we have seen above that developments in the U.S. have been qualitatively and even quantitatively similar for both measures of the labor income share. Karabarbounis and Neiman (2014) have also used this data to document industry-level changes in the labor income share, but have not studied the manufacturing-services division.

We define sectors as for the U.S.. We use countries with at least 15 observations.<sup>15</sup> This

<sup>15</sup>This essentially implies excluding transition economies. We also exclude Korea, because it starts the manufacturing to services transition only part-way through the sample. We also exclude Ireland and Luxembourg due to data problems.

leaves us with 16 countries.

Table 1: The labor income share by sector and country, 1970-2007

country	Manufacturing		Services		Aggregate		SC	N
	level	change	level	change	level	change		
AUS	0.707	-2.5	0.561	-0.8	0.601	-1.9	-0.8	38
AUT	0.719	-6.0	0.589	-4.1	0.633	-5.0	-0.5	38
BEL	0.673	1.0	0.537	3.2	0.581	2.0	-0.7	38
DNK	0.751	-1.7	0.561	2.3	0.615	0.8	-0.4	38
ESP	0.609	0.5	0.584	-4.6	0.587	-3.0	-0.2	38
FIN	0.645	-5.3	0.568	-2.9	0.597	-4.0	-0.2	38
FRA	0.700	0.7	0.611	-4.9	0.632	-3.7	-0.4	38
GER	0.755	-0.9	0.551	-1.1	0.629	-1.8	-1.0	38
GRC	0.748	-3.4	0.410	0.8	0.505	-1.7	-1.6	38
HUN	0.612	-10.9	0.570	2.4	0.584	-2.0	-0.0	13
ITA	0.720	-0.7	0.625	-3.6	0.655	-3.0	-0.4	38
JPN	0.578	-0.1	0.562	-5.1	0.566	-3.1	-0.1	37
NLD	0.680	-2.0	0.644	-2.0	0.654	-2.0	-0.2	38
PRT	0.681	-2.4	0.466	4.5	0.523	2.8	-0.1	37
SWE	0.756	-7.9	0.559	-1.0	0.625	-3.6	-0.4	38
UK	0.758	1.3	0.659	-0.7	0.691	-1.2	-0.8	38

Note: N denotes the number of observations. The level is the sample average for a country. Changes are in units of percentage points per decade. SC = change in the value added share of manufacturing  $\times$  (average LISM – average LISS), also in percentage points per decade. This is how much change in the aggregate labor income share could be explained by structural change alone. The series for Hungary excludes the first three observations (1992-1994), over which the labor income share in manufacturing collapses from 82 to 66% in three years.

The evolution of the LIS in these countries is shown in Table 1. The aggregate labor income share declines in all but three countries, by 2.1 percentage points per decade on average. The labor income share in manufacturing declines in all but 4 countries, by 2.5 percentage points per decade on average. The labor income share in services declines in all but five of the countries in our sample, by 1.5 percentage points per decade on average.<sup>16</sup> Overall trends are thus similar to those observed in the U.S., and even somewhat stronger. The overall ranking of sectors is also similar, with substantially stronger declines of the labor income share in manufacturing.

<sup>16</sup>These trends are statistically significant at the 5% level in all, all but one, and all but two of the cases, respectively.

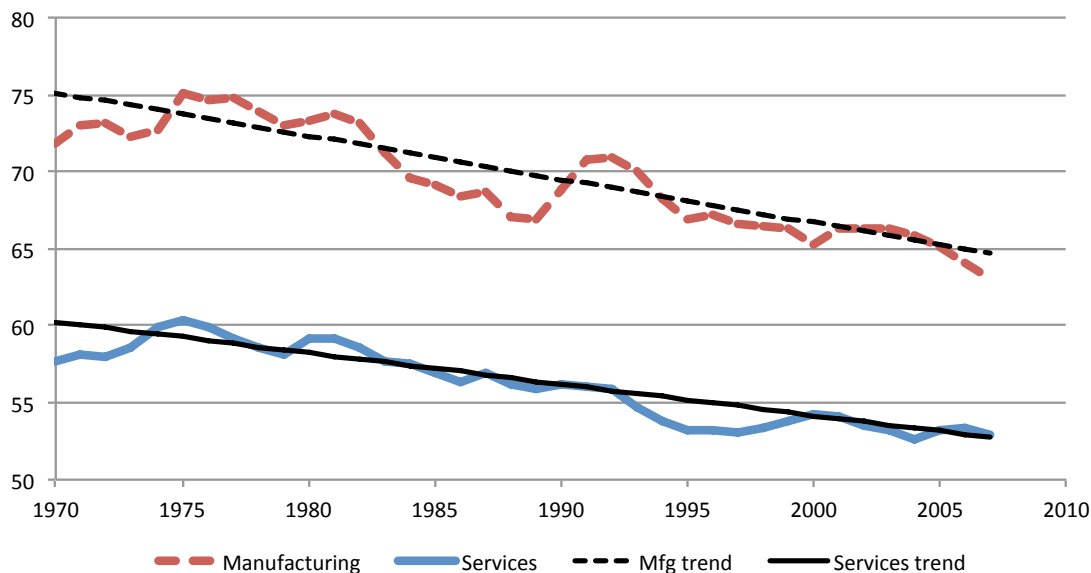
The same pattern is visible more formally in Figure 3, which shows the common component of the labor income share in 17 economies. It is obtained from these two regressions:

$$LIS_{cit} = D_{it} + D_{ic} + \epsilon_{cit} \quad (3)$$

$$LIS_{cit} = \beta_0 year_t + \beta_1 D_i year_t + D_{ic} + \epsilon_{cit}, \quad (4)$$

where  $c, i$  and  $t$  index countries, sectors (manufacturing or services) and time, respectively.  $D_i$  are sector dummies,  $D_{it}$  sector  $\times$  year dummies, and  $D_{ic}$  sector  $\times$  country dummies. From the first specification, we obtain estimates of  $D_{it}$ , or annual sectoral labor income shares net of country-specific effects, which are captured by  $D_{ic}$ . From the second specification, we obtain a time trend for the labor income share in each sector. Figure 3 displays both the series of estimates of  $D_{it}$  and the estimated time trends. It is evident from the figure that labor income shares in both sectors feature a negative trend in our sample, with some cyclical variation. The estimate of the trend coefficient  $\beta_0$  ( $\beta_1$ ) is  $-0.2$  ( $-0.08$ ), with a robust standard error of  $0.024$  ( $0.034$ ).<sup>17</sup> The estimated trend thus implies that on average, the labor income share in services has declined by two percentage points per decade. In manufacturing, it has declined by an additional 0.8 percentage points, or almost 50% more.

Figure 3: The common component of the labor income share in 17 countries, 1970-2007



Note: The figure displays fitted values from the regressions in equations (3) and (4).

<sup>17</sup>Labor income shares enter the regression as percentages. We use data on the 16 countries listed in Table 1 plus the United States. This results in a country by sector panel with 1190 observations and up to 38 observations per country-sector pair. The estimate of  $\beta_0$  is statistically significant at the 1% level, and that of the trend difference  $\beta_1$  at the 5% level.

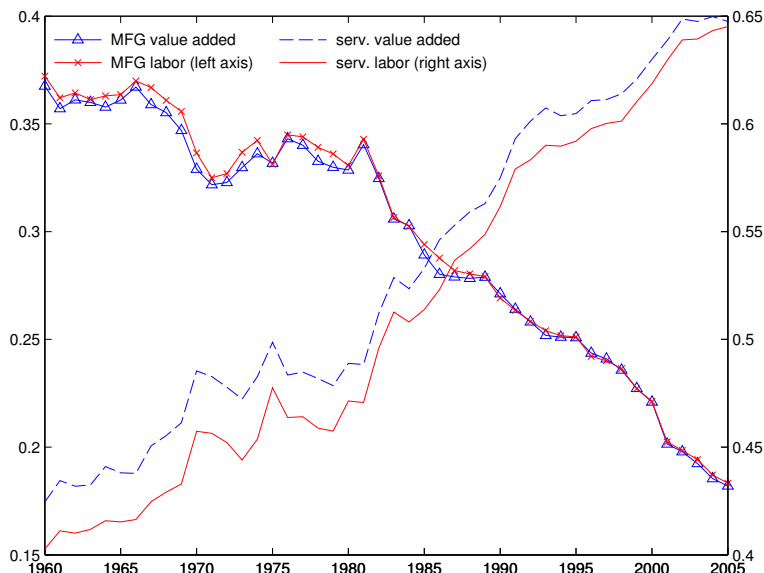
These large changes in sectoral labor income shares account for the bulk of changes in the aggregate labor income share. The penultimate column of Table 1 shows how much change in the aggregate labor income share could be accounted for if sectoral production technologies were Cobb-Douglas. In this case, sectoral factor income shares are fixed at their average value over the sample, and the aggregate labor income share can change only due to shifts in the value added shares of the two sectors, i.e. structural change. The table shows that structural change on its own cannot even account for 10% of the observed change in the labor income share in most countries. (See the next subsection for more information on structural change in the United States.) It could explain more than half of the change in the aggregate labor income share in only three countries, and more than a quarter in only four. Hence, changes in the aggregate labor income share are mostly driven by changes in sectoral labor income shares, and in particular by that in manufacturing, which on average declined about 40% more quickly than that in services.

### 2.3 Structural change in the United States

Structural change, i.e. the reallocation of economic activity across the three broad sectors agriculture, manufacturing and services, has long been known to accompany modern economic growth. (See e.g. Kuznets 1966.) As a consequence, agriculture now employs less than 2% of the workforce in the U.S., while the reallocation between manufacturing and services is still in full swing. Figure 4 shows the evolution of the fraction of labor employed and the fraction of value added produced in the manufacturing and services sectors from 1960 to 2005 using data from Jorgenson’s (2007) 35-sector KLEM data base. Structural change is clearly evident.

Several theoretical channels have been proposed as drivers of structural change, the most prominent being non-homothetic preferences (Kongsamut et al. 2001) on the demand side and differences across sectors in productivity growth (Ngai and Pissarides 2007) or in capital intensity (Acemoglu and Guerrieri 2008) on the supply side. According to the first channel, the rise of the service sector may be due to a higher income elasticity of services demand: in a growing economy, consumers increase the share of their expenditure on services. The second channel, also related to “Baumol’s cost disease”, implies that sectors with rapid productivity growth can shed resources, which are then employed in slower-growing sectors. The third channel is similar in spirit, but the difference lies in the cross-sectoral variation of the contribution of capital to output. Given the patterns of capital intensity and sector-specific technological change, the second channel requires manufacturing and services output

Figure 4: The structural transformation in the United States, 1960-2005



Source: Jorgenson’s (2007) 35-sector KLEM data base.

to be gross complements in consumption, while the third requires the opposite.<sup>18</sup> All these channels can generate structural change in terms of both inputs and outputs. However, as discussed above, none of them can generate the observed movements in factor income shares.

Cross-sectoral differences in the substitutability of capital and labor also have implications for structural change. (We analyze this in detail in the companion paper Alvarez-Cuadrado, Long and Poschke (2017).) Notably, they affect the “shape” of structural change: the extent of capital versus labor reallocation. This occurs because with differences in the elasticity of substitution, the sector with higher substitutability (the “flexible sector”) moves towards using the factor that becomes relatively cheaper more intensively. Hence, if for example the capital bias in technical change exceeds the rate of capital accumulation, then the more flexible sector tends to become more labor-intensive.

<sup>18</sup>It is clear from the figures above that the capital income share in services exceeded that in manufacturing throughout the sample. With competitive factor markets, this implies a larger elasticity of output with respect to capital in services. Productivity growth since World War II, in turn, was larger in manufacturing.

### 3 Theoretical framework: A model with unequal sectoral substitution possibilities

The preceding section documented trends in factor income shares, and showed that changes in factor income shares within sectors, in particular within manufacturing, are key for understanding the decline in the aggregate labor income share. From (2), it is clear that differences in the evolution of sectoral factor income shares can come from differences in (the bias of) productivity growth, differences in factor allocation (differences in the growth of sectoral capital-labor ratios), or differences in the elasticity of substitution between capital and labor. Differences in their level can be due to differences in the capital-intensity of sectors, parameterized by  $\alpha_i$ . All of these factors would also lead to structural change across sectors (Ngai and Pissarides 2007, Acemoglu and Guerrieri 2008, Alvarez-Cuadrado et al. 2017). In addition, non-homothetic demand can also lead to structural change (Kongsamut et al. 2001) and thus to changes in the aggregate labor income share, as just illustrated.

We therefore conduct a joint quantitative analysis of the effect of cross-sectoral differences in capital-labor substitutability and of these additional factors (sectoral differences in productivity growth and in capital intensity, non-homothetic preferences) capable of driving joint changes in the labor income share and structural change. In the next Section, we then calibrate the model to the recent U.S. experience. Since our period of analysis is restricted by the availability of data on properly adjusted sectoral factor income shares from 1960 onwards, we conduct our analysis in an environment with two sectors, manufacturing and services, and abstract from agriculture, which already accounted for only 6% of employment and an even smaller fraction of value added in 1960.<sup>19</sup>

With quantitative results in hand, we can then conduct counterfactual exercises to determine the relative importance of these different factors for changes in the labor income share. As an added benefit, we can also quantify their importance for structural change.

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<sup>19</sup>In terms of sectoral reallocations, the contribution of agriculture over our sample period is similarly small, accounting for barely 10% of the reallocations of labor and 8% of the changes in sectoral composition of value added.



### 3.1 Model setup

We model a closed economy in discrete time. It is populated by a representative infinitely-lived household with instantaneous preferences given by<sup>20</sup>

$$v = \ln(u(c_m, c_s)) \quad \text{where} \quad u(c_m, c_s) = \left( \gamma c_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_s + s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The subscripts  $m$  and  $s$  denote manufactures and services, respectively, so  $c_i$  stands for per capita consumption of value-added produced in sector  $i$ ,  $i \in \{m, s\}$ .  $\varepsilon$  is the elasticity of substitution between the two consumption goods. The term  $s$  introduces a non-homotheticity. The empirically relevant case is  $s > 0$ , which can be interpreted as households having an endowment of services. As a consequence, the income elasticity of demand for services is larger than that for manufactures. This introduces the possibility of demand-driven structural change, as in Kongsamut et al. (2001).

Sectoral outputs are produced according to general CES technologies.

$$Y_m = A_m \left[ \alpha_m (\tilde{B}_m K_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m) L_m^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} \quad (5)$$

$$Y_s = A_s \left[ \alpha_s (\tilde{B}_s K_s)^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s) L_s^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}} \quad (6)$$

This structure allows for a rich set of asymmetries. First, we allow for sector-specific elasticities of substitution and distributional parameters,  $\sigma_i$  and  $\alpha_i$  respectively. Second, we assume that the sectoral level of Hicks-neutral productivity,  $A_i$  and the factor imbalance of technical change,  $\tilde{B}_i$ , grow at constant exponential rates  $g(A_i)$  and  $g(\tilde{B}_i)$ , respectively, and allow both their initial levels and these growth rates to differ across sectors. As a result, the bias in technical change,  $\frac{(1-\sigma_i)}{\sigma_i} g(\tilde{B}_i)$ , may also differ across sectors.<sup>21</sup> Third, non-homothetic preferences introduce an additional difference between the services and manufacturing sectors. Finally, we follow most of the literature on multi-sector models by assuming that capital is only produced in the manufacturing sector, so services are fully consumed. Therefore, using upper-case letters to denote aggregate variables,  $Y_s = C_s$  and  $Y_{m,t} = C_{m,t} + K_{t+1} - (1-\delta)K_t$ , where  $\delta > 0$  is the constant rate of depreciation of the capital stock.

<sup>20</sup>Here and in the following, we omit time subscripts except in expressions that involve variables dated at different points in time.

<sup>21</sup>Diamond, McFadden and Rodriguez (1978) show that in the presence of biased technical change, data on output, inputs, and input prices are not sufficient for the identification of the elasticity of substitution. In addition, functional restrictions on the rate of technical change are required. A constant exponential growth rate, as assumed here, is the standard identification restriction.

Factors are fully utilized which, normalizing the labor endowment to one, implies

$$\frac{L_s}{L} + \frac{L_m}{L} \equiv l_m + l_s = 1 \quad (7)$$

$$l_m k_m + l_s k_s = l_m k_m + (1 - l_m) k_s = k, \quad \text{where } k_i \equiv \frac{K_i}{L_i}. \quad (8)$$

In the following, we will use lower-case variables to denote per capita quantities, except for  $k_i$ , which stands for the capital-labor ratio in sector  $i$ .

### 3.2 Model solution

We choose manufactures to be the numeraire and denote the price of services by  $p_s$ . Since markets are competitive, production efficiency requires equating marginal revenue products across sectors, so

$$\alpha_m \left( A_m \tilde{B}_m \right)^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{K_m} \right)^{\frac{1}{\sigma_m}} = p_s \alpha_s \left( A_s \tilde{B}_s \right)^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{K_s} \right)^{\frac{1}{\sigma_s}} = R \quad (9)$$

$$(1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{L_m} \right)^{\frac{1}{\sigma_m}} = p_s (1 - \alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{L_s} \right)^{\frac{1}{\sigma_s}} = w. \quad (10)$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left( \frac{1 - \alpha_m}{1 - \alpha_s} \frac{\alpha_s}{\alpha_m} \tilde{B}_m^{\frac{1-\sigma_m}{\sigma_m}} \right)^{\sigma_s} \tilde{B}_s^{\sigma_s-1} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \quad (11)$$

Using this notation, (9) implies that the relative price of services is given by

$$p_s = \frac{\alpha_m A_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) k_m^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}{\alpha_s A_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (\varrho k_m^{\frac{\sigma_s}{\sigma_m}})^{\frac{1-\sigma_s}{\sigma_s}} \right]^{\frac{1}{\sigma_s-1}}}. \quad (12)$$

Household optimization in turn requires equating the marginal rate of substitution between the two consumption goods to their relative price in every period, implying relative demands given by

$$\frac{v_s}{v_m} = \frac{(1 - \gamma)}{\gamma} \left( \frac{c_s + s}{c_m} \right)^{-\frac{1}{\varepsilon}} = p_s. \quad (13)$$

Since output in services is fully consumed, the sectoral labor allocation is given by

$$l_m = 1 - p_s c_s \frac{\alpha_s}{\alpha_m} \frac{\tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}}}{A_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}}} \frac{\left[ \alpha_m \left( \tilde{B}_m k_m \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) \right]^{\frac{1}{1-\sigma_m}}}{\left[ \alpha_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} \varrho k_m^{\frac{\sigma_s-1}{\sigma_m}} + (1 - \alpha_s) \varrho^{\frac{1}{\sigma_s}} \right]} \quad (14)$$

Given a solution to the dynamic problem and state variables  $k, A_m, A_s, \tilde{B}_m, \tilde{B}_s$ , equations (11), (12) and (14) pin down  $p_s, k_m$  and  $l_m$  at each point in time. Their counterparts for services,  $k_s$  and  $l_s$ , then follow from equations (7) and (8).

The law of motion of per capita capital is given by

$$(1+n)k_{t+1} = y_{m,t}f_m(k_{m,t})l_{m,t} - c_{m,t} + (1-\delta)k_t, \quad (15)$$

where

$$y_{m,t} \equiv f_m(k_{m,t})l_{m,t} = A_{m,t} \left[ \alpha_m \left( \tilde{B}_{m,t}k_{m,t} \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m) \right]^{\frac{\sigma_m}{\sigma_m-1}} l_{m,t},$$

that combined with (8) and (11) determines the law of motion of  $k_m$ .

The solution to the household's dynamic problem implies the Euler equation

$$c_{t+1} = \frac{\beta}{1+n} [f'_m(k_{m,t+1}) + 1 - \delta] (c_t + p_{s,t}s) - p_{s,t+1}s. \quad (16)$$

where  $\beta$  is the subjective discount factor,  $n$  is the rate of population growth, and  $c$  is consumption expenditure,  $p_s c_s + c_m$ .

An equilibrium in this economy consists of sequences of allocations  $\{c_m(t), c_s(t), k_m(t), k_s(t), k(t), l_m(t), l_s(t)\}_{t=0}^{\infty}$  and prices  $\{p_s(t), w(t), R(t)\}_{t=0}^{\infty}$  such that equations (9) to (16) hold and feasibility is satisfied.

Before proceeding to our quantitative results, three technical remarks are in order. First, since the marginal product of capital in manufacturing in the future depends on the labor allocation in that future period, the dynamic problem is not independent of the static problem.<sup>22</sup> Hence, we cannot separate the system of equations (14), (15) and (16) into two blocks, but need to solve all three equations together.

Second, our model allows for Hicks-neutral technical change which, as is well-known, is not consistent with balanced growth. In general, many multi-sector models are not consistent with balanced growth. This is true for Kongsamut et al.'s (2001) model of demand-driven structural change, except in a special case, and also for Acemoglu and Guerrieri's (2008) model of technology-driven structural change, except in the limit. This is no coincidence. In their chapter on structural change in the Handbook of Economic Growth, Herrendorf, Rogerson and Valentinyi (2014) state (p. 4): "It turns out that the conditions under which one can simultaneously generate balanced growth and structural transformation are rather strict, and that under these conditions the multi-sector model is not able to account for the

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<sup>22</sup>This does not depend on whether we use the law of motion of  $k$  or  $k_m$  in the dynamic problem. Key is that the future marginal product of capital in manufacturing that appears in the Euler equation (16) depends on the future labor allocation.

broad set of empirical regularities that characterize structural transformation. ... we think that progress in building better models of structural transformation will come from focusing on the forces behind structural transformation without insisting on exact balanced growth.” As the quantitative results below show, our model features approximate balanced growth.

Third, the absence of exact balanced growth requires the imposition of a different terminal condition for the dynamic system given in equations (14) to (16). We proceed by a) choosing a finite time horizon  $T$  (100 years in the results shown below), b) imposing that consumption growth is constant at the end of that horizon:  $c_{T+1}/c_T = c_T/c_{T-1}$  (this implies a solution to equation (16) for  $t = T$ , which in turn allows solving equations (14) and (15) for that period), and c) check that the specific horizon  $T$  that we chose does not affect results.

## 4 Quantitative analysis: Dissecting the U.S. experience

In this section, we describe the calibration of the model and present a set of counterfactual exercises decomposing model-predicted changes in sectoral labor income shares and in structural change into four components associated with the four fundamental drivers of sectoral differences in the model. These will show the relative contribution of the different fundamental forces to the observed declines in labor income shares.

### 4.1 Calibration

#### 4.1.1 Data

We calibrate the model to U.S. data over the period 1960-2005. The data we use to obtain factor income shares and allocations is from Jorgenson (2007) and has been discussed above. We convert capital services reported there into capital stock figures using the average long-run rental rate from the model. To calibrate preferences, we also require information on consumption shares and the relative price, which we take from Herrendorf, Rogerson and Valentinyi (2013).

#### 4.1.2 Preferences

Herrendorf et al. (2013, Figures 9 and 10) show that despite an increase in the relative price of services, the ratio  $c_s/c_m$  has not fallen, but increased slightly over the last 65 years. With  $s \geq 0$ , Leontief preferences between manufacturing and services ( $\varepsilon = 0$ ) provide the

best approximation to this trend. This is, of course, in line with Herrendorf et al.'s (2013) estimates.<sup>23</sup>

Given  $\varepsilon = 0$ , equation (13) allows us to calibrate the non-homotheticity parameter  $s$  using the growth rates of quantities and prices of manufacturing and services consumption, in value added terms, over the 1960 to 2005 period. This results in a value for  $s$  of around 20% of first period services consumption. We then obtain a value for  $\gamma$ , the weight of manufacturing goods in preferences, from the relative weight of manufacturing and services consumption in the initial period. Since the ratio of manufacturing to services consumption does not change much, this value is not very sensitive to the time period we use for calibrating it.

### 4.1.3 Technology

For reference, each sector's CES production function is given by

$$Y_{it} = A_{it} \left[ \alpha_i (\tilde{B}_{it} K_{it})^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \alpha_i) L_{it}^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}, \quad (17)$$

where  $\alpha_i, \sigma_i, A_{i0}, \tilde{B}_{i0}$ , and the growth rates of  $A_{it}$  and  $\tilde{B}_{it}$  are all allowed to differ across sectors. In general, changing  $\sigma$  in (17), as we do for counterfactuals below, does not only change the substitution elasticity (see e.g. Klump et al. 2007). We therefore work with a normalized version of the CES recommended by León-Ledesma et al. (2010):

$$Y_{it} = \bar{Y}_i \frac{A_{it}}{\bar{A}_i} \left[ \bar{\theta}_i \left( \frac{\tilde{B}_{it} K_{it}}{\tilde{B}_i \bar{K}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} + \overline{1 - \theta}_i \left( \frac{L_{it}}{\bar{L}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}. \quad (18)$$

Here, variables with a bar denote the geometric sample average of the underlying series, and  $\bar{\theta}_i$  and  $\overline{1 - \theta}_i$  refer to the average sectoral capital and labor income shares, respectively. In terms of the production function above, using this normalization involves setting  $\alpha_i$  to  $\bar{\theta}_i$  and  $1 - \alpha_i$  to  $\overline{1 - \theta}_i$  (the two terms sum approximately to one even with geometric averages of the income shares). Since  $\alpha_i, A_{it}$  and  $\tilde{B}_{it}$  in (17) cannot be identified separately, choosing the value of  $\alpha_i$  in this way is a normalization. Herrendorf et al. (2015) use the same normalization when estimating sectoral production functions.<sup>24</sup> The normalization chosen matters for exercises where the substitution elasticity is changed. The one we use implies that for all  $\sigma_i$ , isoquants are tangential at the average capital-labor ratio in the data, and

<sup>23</sup>In Appendix C, we show that results are very similar when calibrating the model with a larger  $\varepsilon$  of 0.5; the route taken by Buera and Kaboski (2009) to avoid Leontief preferences.

<sup>24</sup>On normalization of CES production functions, see León-Ledesma et al. (2010) and Cantore and Levine (2012). Temple (2012) discusses the interpretation of the normalized production function in detail.

output at that capital-labor ratio does not change with  $\sigma_i$ . With this normalization, changing  $\sigma_i$  (for example when computing a counterfactual to evaluate the effect of differences in  $\sigma$  across sectors) thus does not affect sectoral output at the sample average.

It is clear that even once the distributional parameter is fixed in this way, levels of  $A_{it}$  and  $\tilde{B}_{it}$  are not identified independently of the units of  $Y_{it}$ . We therefore normalize  $A_{i0}\tilde{B}_{i0}(= B_{i0})$  to 1 in each sector. We also set the predetermined initial capital-labor ratio in manufacturing from the data.<sup>25</sup> We set the depreciation rate  $\delta$  to 5% per year and the discount factor  $\beta$  to 0.94.

At this point, eight parameters remain to be calibrated: the two elasticities of substitution between capital and labor,  $\sigma_m$  and  $\sigma_s$ , four growth rates of  $A_m, A_s, \tilde{B}_m, \tilde{B}_s$ , and initial levels  $A_{m0}$  and  $A_{s0}$ . We set their values to match eight data moments, all computed for the period 1960 to 2005: the change in the capital income share to labor income share ratio in each sector, the average labor income share in each sector, the change in the fraction of labor and capital (respectively) employed in manufacturing, the average fraction of labor employed in manufacturing, and the aggregate growth rate of output per worker, measured in units of the aggregate consumption good. All these data moments are computed from Jorgenson's (2007) data.<sup>26</sup>

We choose these targets because of their information content regarding the model parameters. While all eight remaining parameters affect the values of all targets, some relationships are particularly strong. The substitution elasticities  $\sigma_i$  and the factor imbalance in productivity growth,  $g(\tilde{B}_i)$ , drive changes in relative income shares in a sector as well as the shape of structural change (strength of labor versus capital reallocation). The relative growth rates of  $A_m$  and  $A_s$ ,  $g(A_m/A_s)$ , contribute strongly to the pace of structural change. The growth rate of  $A_m$  then determines overall output growth.<sup>27</sup> And finally, given our normalization of initial technology levels, the values of  $A_{m0}$  and  $A_{s0}$  determine  $\tilde{B}_{m0}$  and  $\tilde{B}_{s0}$ , and thus strongly affect the average factor income shares in the two sectors.<sup>28</sup>

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<sup>25</sup>One way to understand the need to impose initial conditions on the sectoral allocations is to contrast our solution method with that of a two sector model with a well-defined steady state. In this latter case, the initial conditions for sectoral allocations are determined in such a way that the system eventually converges to its steady state. In our case, in the absence of a steady state, we pin down the initial values of the sectoral allocations from the data. Thereafter, these allocations are endogenously determined. This is also the approach followed by Acemoglu and Guerrieri (2008).

<sup>26</sup>To avoid excessive influence of the first and last observation, we compute the growth rates between averages for the first and last 5 years.

<sup>27</sup>The Jacobian of model moments with respect to parameters, evaluated at our selected parameter values, has full rank.

<sup>28</sup>This is intuitive once you consider that an alternative normalization consisted in setting  $A_{i0}$  to 1 in each sector and calibrating  $\alpha_i$  to match the average labor income share in sector  $i$ . Moreover, it is important to realize that with CES production functions, setting  $\theta_i$  to equal observed factor income shares does not on its

## 4.2 Calibration results

Calibration results are shown in Table 3. The model can reproduce both the differential pattern in changes in the labor income share shown in Figure 2 and the amount and shape of structural change observed in the data. It also matches average levels of the labor income shares and the labor allocation well, and fits aggregate output growth exactly. It does not fit the average labor income share in services exactly, as there is a tension between doing so and fitting the average labor allocation. In addition, the model replicates very closely the average and changes in the capital and output allocations, which were not targeted in the calibration. As a consequence, it also replicates the level and changes of the aggregate labor income share rather closely.

Table 2: Calibration: Parameters.

<hr/>			
<i>general:</i>			
$\delta$ (depreciation rate)			0.05
$\beta$ (discount factor)			0.94
$\varepsilon$ (elasticity of substitution in consumption)			0
$\gamma$ (weight on manufacturing good in consumption)			0.314
<hr/>			
<i>sector-specific:</i>	Manufacturing	Services	Aggregate (implied)
growth of $A_i$ (neutral technology, % p.a.)	7.8	1.1	2.1
growth of $\tilde{B}_i$ (factor imbalance in technical change, % p.a.)	-9.4	-7.0	-2.7
overall sectoral technical change (implied, % p.a.)	4.6	-1.3	1.2
$\sigma_i$ (capital/labor elasticity of substitution)	0.776	0.571	0.64
$A_{i0}$ (initial level of neutral technology)	0.604	0.709	
$\theta_i$ (normalized weight on capital in production)	0.333	0.362	
<hr/>			

Calibrated parameters are shown in Table 2. Key parameters coming from the calibration are those for the sectoral substitution elasticities and the growth rates. The table also shows the implied aggregate equivalents, which we derive below. The calibration yields substitution elasticities below but close to unity in manufacturing, and substantially below unity in services. Our results suggest positive rates of Hicks-neutral productivity growth in

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own imply that the model will match these shares. For this to occur, it is necessary to set  $A_{i0}$  appropriately, given the chosen normalization of the distributional parameter.

Table 3: Calibration: Targets and model moments.

	Data			Model		
	Manufacturing	Services	Aggregate	Manufacturing	Services	Aggregate
<i>calibration targets:</i>						
average sectoral LIS (%)	66.7	63.8		66.4	65.1	
growth in sectoral KIS/LIS (% p.a.)	0.6	0.3		0.6	0.3	
change in manufacturing employment share	-22.4			-23.9		
change in manufacturing share of capital	-21.3			-20.7		
average manufacturing employment share	34.2			34.5		
growth in output per worker (% p.a.)			1.8			1.8
<i>not targeted:</i>						
average aggregate LIS (%)			64.8			65.6
growth in aggregate KIS/LIS (% p.a.)			0.5			0.4
average manufacturing share of capital	33.2			33.3		
average manufacturing value added share	35.1			34.1		
change in manufacturing value added share	-24.5			-22.9		

Note: The data period used is 1960 to 2005. LIS (KIS) stands for the labor (capital) income share. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.



both sectors, with a much larger growth rate in manufacturing. This is combined with a strong capital bias in technical change ( $\frac{1-\sigma_i}{\sigma_i}g(\tilde{B}_i) < 0$ ) in both sectors.

#### 4.2.1 Understanding calibration results

To understand what drives the calibration results for the sectoral elasticities of substitution  $\sigma_i$  and for the sectoral rates of technical change, it is useful to consider how the model can match observed patterns in sectoral factor income shares and the shape of structural change. The key observations here are that in the data, the labor income share declines in both sectors, and more so in manufacturing, and that structural change looks similar, whether expressed in terms of the allocation of employment or that of capital. Given limited importance of non-homotheticity and  $\alpha$  differences (see the decomposition below), the amount and shape of structural change and the relative evolution of the labor income shares in the model depend on the substitution elasticities and on productivity growth rates in the two sectors.

Consider first the evolution of factor income shares. This is determined by equation (2), which is reproduced here for convenience:

$$\frac{KIS_{it}}{LIS_{it}} = \frac{\alpha_i}{1 - \alpha_i} \left( \tilde{B}_{it} \frac{K_{it}}{L_{it}} \right)^{\frac{\sigma_i - 1}{\sigma_i}}$$

From here it is clear that if  $\sigma_i < 1$  and capital per worker grows in both sectors, technical change needs to be capital biased (i.e.  $\tilde{B}$  needs to decline) in both sectors for the labor income share to decline in both of them. The decline in the labor income share thus is driven by the increasing scarcity of effective capital relative to effective labor,  $\tilde{B}K/L$ , which raises the marginal product of capital relative to that of labor. The larger  $\sigma_i$ , the more negative  $g(\tilde{B}_i)$  needs to be to induce a given decline in the labor income share. In line with this,  $|g(\tilde{B}_m)| > |g(\tilde{B}_s)|$  in our calibration, given that  $\sigma_m > \sigma_s$ .<sup>29</sup>

To illustrate how the combinations of the factor imbalance of technical change and the substitution elasticity in each sector are pinned down by the calibration, we consider what would happen if the factor imbalance  $g(\tilde{B}_i)$  was weaker. For a smaller value of  $\sigma_i$ , this could still allow for a similar change in the factor income share in a sector, as is evident from equation (2). To answer this question, we reduce  $\sigma_m$  and the absolute value of  $g(\tilde{B}_m)$  in such a way as to keep the change in the labor income share and the growth of output per

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<sup>29</sup>A rising capital income share is also qualitatively consistent with  $\sigma > 1$  and rising  $\tilde{B}K/L$ , as in Karabarbounis and Neiman (2014). In our calibration, we also searched the region with  $\sigma_i > 1$  but could not find parameter combinations with a good fit to the data. In this region, it turns out to be much harder than in the benchmark to fit both the average labor income share in services and the average labor allocation. Parameterizations with a reasonable  $LIS_s$  feature far too little structural change, in terms of labor, capital and value added.

worker in manufacturing unaffected. Given a background of increasing scarcity of effective capital relative to effective labor, as required to match the evolution of factor income shares, reducing  $\sigma_m$  makes manufacturing less flexible and thus pushes firms in the sector to retain more capital. At the same time, a weaker factor imbalance reduces the speed at which capital becomes more scarce, and thus pushes firms in the other direction. Quantitatively, the first channel dominates, making manufacturing progressively more capital-intensive. This leads to a worse fit in terms of the shape of structural change, with too little change in the fraction of capital used in manufacturing. With our calibrated parameters, in contrast, these forces are balanced in the right way. This example shows how information on structural change helps identify parameters of the model.

The large biases in technical change, very different growth rates of Hicks-neutral technology, and values of  $\sigma_i$  reported in Table 2 are thus what is required to simultaneously match the observed changes in sectoral factor income shares and structural change. Estimates of a capital bias in technical change are common in the literature (see for instance Antras 2004, Herrendorf et al. 2015). This bias is in line with the implications of theories of directed technical change, in particular Acemoglu (2002), who explores the incentives of profit-seeking innovators to develop capital- or labor-augmenting technologies. These incentives are shaped by two competing effects: the price effect, which directs innovation towards the more expensive factor, and the market size effect, which encourages the development of technologies that augment the more abundant factor. The resulting overall direction of technical change depends on the elasticity of substitution between the two factors. When capital and labor are gross complements, the secular increase in the capital-labor ratio raises the wage to rental ratio more than proportionally. That is, the price effect dominates, increasing the incentive to develop labor-augmenting technologies. In terms of our notation,  $g(\tilde{B}) < 0$ . With gross complements, this factor imbalance in technical change implies that the marginal product of capital rises more than that of labor, i.e. technical change is capital-biased.<sup>30</sup>

The overall sectoral productivity growth rates implied by our calibration also merit a remark. Following Diamond, McFadden and Rodriguez (1978), define overall technical progress in a sector as growth in sectoral output per worker net of the effect of capital deepening,  $g(Y_i/L_i) - \epsilon_i g(K_i/L_i) = g(A_i) + \epsilon_i g(\tilde{B}_i)$ , where  $\epsilon_i$  is the elasticity of sectoral output with respect to capital, which coincides with the capital income share when factor markets are

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<sup>30</sup>Specifically in Acemoglu (2002) the relative profitability of new capital-augmenting technologies,  $B$ , relative to labor-augmenting technologies,  $A$ , is proportional to  $(\frac{B}{A})^{-\frac{1}{\sigma}} (\frac{K}{L})^{\frac{\sigma-1}{\sigma}} \equiv \tilde{B}^{-\frac{1}{\sigma}} (\frac{K}{L})^{\frac{\sigma-1}{\sigma}}$ . When  $\sigma < 1$ , increases in  $\frac{K}{L}$  imply that the relative profitability of capital-augmenting innovations declines. To equilibrate innovation incentives,  $\tilde{B}$  has to rise. This is capital-biased technical change, i.e.  $\frac{(1-\sigma)}{\sigma} g(\tilde{B}) > 0$ .

competitive. Inserting calibrated parameters and model outcomes implies growth rates of overall technology of 4.6% per year in manufacturing and -1.3% in services. Faster overall technical change in manufacturing compared to services is in line with other sources (e.g. Herrendorf et al. 2015). Negative rates of technical change in some service sectors, for example utilities, are also frequently found in the literature on productivity measurement (see in particular Harper, Khandrika, Kinoshita and Rosenthal 2010, Jorgenson, Ho and Samuels 2016) and are typically attributed to regulation and/or measurement difficulties.

It is less usual to find a negative growth rate for the entire service sector. This finding can be attributed to our identification strategy, where the difference in productivity growth between sectors is mostly identified from the extent of structural change in the data. As a consequence, the model requires a certain productivity growth difference between sectors to generate the observed amount of structural change. The required growth difference is fairly large, and combined with an overall output growth rate of 1.8% implies negative growth in the slow-growing sector (services). It is important to realize that this is not due to the setting with CES production, but is also required for a model with Cobb-Douglas sectoral production functions to match the observed amount of structural change. (Calibration results for such a model are shown in Tables 7 and 8 in Appendix C.2.) Targeting a (counterfactually) lower amount of structural change or allowing for an additional driver of structural change would result in more similar rates of technical progress in the two sectors, allowing the rates to be positive in both sectors. Other potential drivers of structural change, from which we abstract here, include scale economies (Buera and Kaboski 2012*b*), specialized high-skilled labor (Buera and Kaboski 2012*c*), and trade (Swiecki 2017). We leave exploration of a combined model involving further channels for future research.

We conclude that the key objects for understanding the evolution of factor income shares – the substitution elasticities and the bias of technical change – are mostly identified from the shape of structural change, and not the overall amount.

#### 4.2.2 Comparison to other estimates of substitution elasticities

Next, we briefly compare our calibrated values for the substitution elasticities  $\sigma_i$  to estimates from the literature. Estimates of substitution elasticities below one are in line with previous estimates at the aggregate level compiled in León-Ledesma et al. (2010). For the sectoral level, we are only aware of estimates from Oberfield and Raval (2014) (for manufacturing only) and from Herrendorf et al. (2015). Our estimate of  $\sigma_m$  lies right in between the ones in these two papers, which are 0.7 and 0.8. Herrendorf et al.’s (2015) estimate for services is

0.75. While this is slightly higher than ours, it also implies that manufacturing is the more flexible sector. Calibrating all parameters jointly, instead of estimating sector by sector, thus makes a difference.<sup>31</sup>

To compare our results to the broader literature estimating the elasticity of substitution at the aggregate level, we can compute an analogous object in the model economy. It is given by

$$\sigma = \gamma_0 \varepsilon + \gamma_1 \sigma_s + \gamma_2 \sigma_m + \frac{(l_s - \kappa_s)}{(g(w) - g(R))} \left( g(Y_m) - n - g(c_m) \left( 1 + \frac{s}{c_s} \right) \right), \quad (19)$$

where

$$\begin{aligned} \gamma_0 &\equiv (\epsilon_m - \epsilon_s) (l_s - \kappa_s) \left( 1 + \frac{s}{c_s} \right), \\ \gamma_1 &\equiv l_s \epsilon_s + \kappa_s (1 - \epsilon_s), \\ \gamma_2 &\equiv (1 - l_s) \epsilon_m + (1 - \kappa_s) (1 - \epsilon_m), \end{aligned}$$

and  $l_s \equiv L_s/L$  and  $\kappa_s \equiv K_s/K$  are the fractions of labor and capital used in the service sector, respectively. (See Appendix B.2 for a derivation of these expressions.)

The aggregate elasticity of substitution between capital and labor is a weighted average of production elasticities of substitution in the two sectors and the elasticity of substitution in consumption between manufacturing and services output. The final term in (19) is due to the modeling assumption that investment goods are only produced in the manufacturing sector. As the weights in (19) depend on the allocation of inputs, the aggregate elasticity is time-varying. In our benchmark economy, it declines from 0.65 to 0.63 as the weight of manufacturing, the more flexible sector, declines. The sample average is 0.64. Hence, it is close to estimates of the aggregate elasticity in the literature. For instance, León-Ledesma et al. (2010, Table 1) report many estimates between 0.5 and 0.8, Herrendorf et al. (2015) find a value of 0.85, and Chirinko and Mallick (2017) a value of 0.4.<sup>32</sup>

Using the aggregate elasticity of substitution just computed, we can also compute measures of aggregate technical change. We do so in two steps. First, using the aggregate version of equation (2) combined with model data on aggregate factor income shares, capital deepening, and the aggregate elasticity of substitution yields a measure of the aggregate bias of

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<sup>31</sup>When inserting their estimates in our model, the model predicts too much change in the labor income share in manufacturing, and too little (in fact, positive) change in services. The latter is due to the higher substitution elasticities in services estimated by Herrendorf et al. (2015). The model also predicts almost no change in the share of capital employed in manufacturing.

<sup>32</sup>Our estimates would be slightly closer to those of Herrendorf et al. (2015) if our model included agriculture, where the elasticity of substitution is generally estimated to be above 1.

technical progress,  $\frac{\sigma-1}{\sigma}g(\tilde{B})$ .<sup>33</sup> Second, we again compute aggregate technical progress as growth in output per worker net of the effect of capital deepening, i.e.  $g(Y/L) - \epsilon g(K/L)$  or  $g(A) + \epsilon g(\tilde{B})$ . This implies average annual values of  $g(A)$  of 2.1% and of  $g(\tilde{B})$  of -2.7%. The former is very close to estimates in Antras (2004), Herrendorf et al. (2015) and León-Ledesma et al. (2015). Aggregate  $g(\tilde{B})$  in our model economy is slightly less negative than estimated by Herrendorf et al. (2015) and Antras (2004), who find values around -3%, and about one percentage point more negative than the estimates of León-Ledesma et al. (2015).

Overall, measures of aggregate technical change are less extreme than sectoral ones. (This pattern is also visible in Herrendorf et al.’s (2015) estimation results.) This arises for two reasons. First, aggregate growth rates combine changes in the two sectors. Second, they are in units of final output, whereas sectoral growth rates are in units of sectoral output.

### 4.2.3 The benchmark time path

Figure 5 compares the time path of our simulated economy with the data. The top right panel shows output per capita, which grows at an average and almost-constant rate of 1.8% per year. The interest rate (top left panel) is also almost constant: after an initial transition of 18 years, during which it increases by 1.3 percentage points, it becomes almost constant, falling only 0.11 percentage points over the remaining 28 years of our simulated data. The economy thus exhibits “approximate balanced growth”.<sup>34</sup>

The two bottom panels show structural change and the change in factor income shares. The left panel shows that the labor income share declines in both sectors, but more sharply in manufacturing, driven by the larger factor imbalance of productivity growth in that sector. Since this is a calibration target, the model reproduces the observed declines in the labor income share well and also matches their average level rather well. The aggregate labor income share declines from 68.2% to 63.8% in the model, compared to a fall from 68.4% to 63.0% in the data. While this moment was not targeted directly, it is clearly closely related to three of our calibration targets.

The right panel depicts structural change generated in the model. It shows that the calibrated model fits the data both in terms of the amount of structural change generated and in terms of its composition: the reallocation of labor and capital look strikingly similar

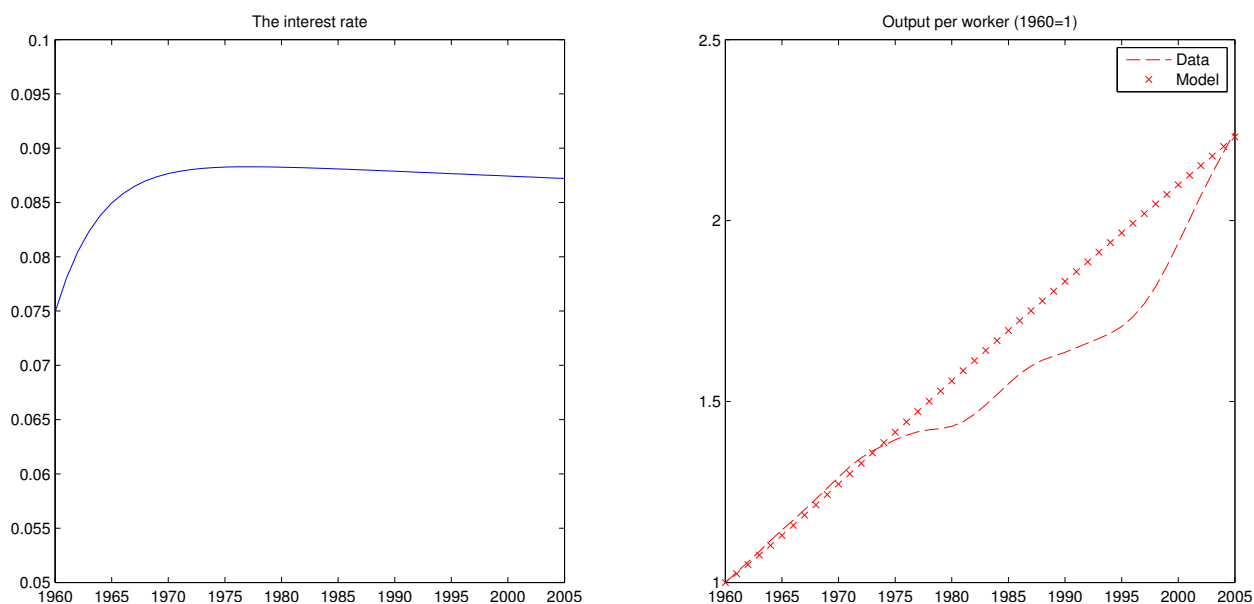
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<sup>33</sup>Note that this exercise implies assuming that the aggregate model data is generated by an aggregate production function with a constant elasticity of substitution. This of course runs counter to the two-sector model we use. The numbers we give here therefore purely serve to illustrate what an econometrician who used our model data and made this assumption would conclude.

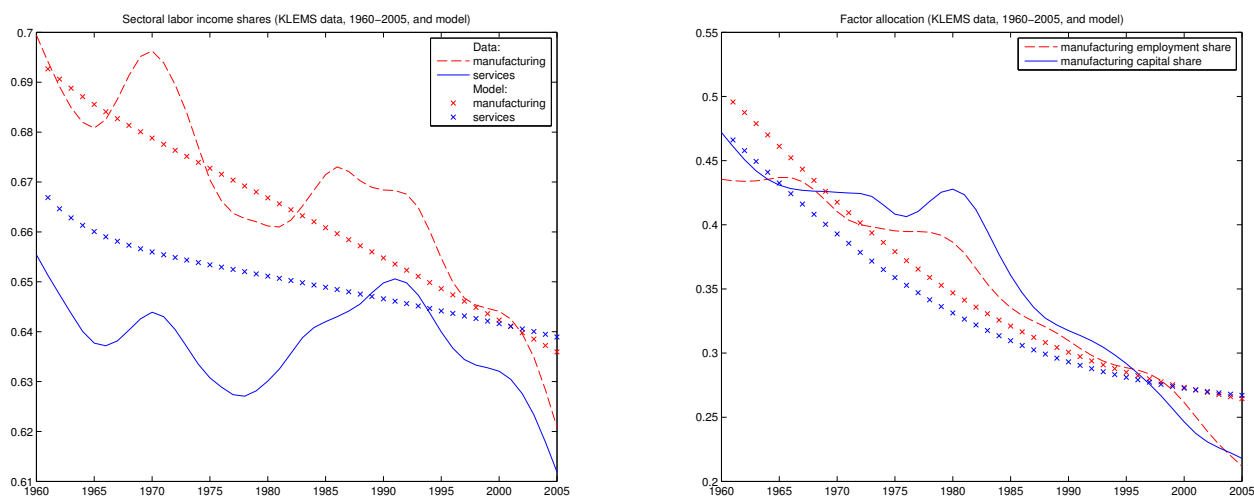
<sup>34</sup>The initial capital stock appears to be too high. This is similar to Acemoglu and Guerrieri (2008), who find a small decline in the interest rate in the first years, followed by stabilization.

both in the model and in the data, apart from some medium-run fluctuations which the model is not designed to capture. The model also fits the data well in terms of the level and change of the share of output in manufacturing; it falls by 22.9 percentage points, versus 24.5 in the data.<sup>35</sup>

Figure 5: The benchmark economy



(a) The rental rate and aggregate output per capita



(b) Factor income shares and structural change in the benchmark economy

Notes: Parameters are given in Table 3. Data is smoothed using an HP-Filter with smoothing parameter 6.25.

<sup>35</sup>Note that this was not targeted. Since the output share depends not only on the input shares but also on productivity and prices, it is not obvious that it should fit so well.

Overall, we take the fact that, despite some differences in the data used, our calibration results are close but not identical to findings in the related literature as a vindication of our approach. The literature on the estimation of CES production functions argues that using more than a single equation in estimation alleviates identification difficulties. It turns out that the effect on identified parameter values coming from the additional restrictions imposed by our approach compared to for instance the estimation in Herrendorf et al. (2015) is moderate. The main benefit of using a full dynamic model comes into play in the decomposition exercise, to which we turn next.

### 4.3 Decomposition

In the following, we assess the relative importance of different drivers on changes in labor income shares. To do so, we explore how results change once we separately disable each one of the four fundamental drivers in the model.

To eliminate the non-homotheticity, we set  $s$  to zero. To eliminate differences in capital intensity, we set  $\bar{\theta}_m$  and  $\bar{\theta}_s$  to the average of the two, keeping  $A_{m0}$  and  $A_{s0}$  as in the benchmark. To eliminate differences across sectors in the factor imbalance of productivity growth, we set  $g(\tilde{B}_i)$  in each sector to its average value in the two sectors, while adjusting  $g(A_i)$  in both sectors such that aggregate output growth equals that in the benchmark economy. To eliminate differences across sectors in the levels of productivity growth, we set  $g(A_m) = g(A_s)$  and  $g(\tilde{B}_m) = g(\tilde{B}_s)$  and choose growth rates such that aggregate output growth and the mean factor imbalance remain unchanged. Finally, to eliminate differences in  $\sigma_i$ , we set  $\sigma$  in both sectors to the average, 0.673.

Decomposition results are reported in Table 4. The table shows the effect of these changes both on changes in sectoral and aggregate labor income shares and on structural change. All changes worsen the fit of the model, though only slightly for setting  $s$  to zero.

It is clear that two channels, non-homothetic preferences and differences in capital intensity, hardly matter. Setting  $s$  to zero hardly affects structural change, and therefore hardly affects labor income shares. The reason for this is that with low  $s$ , the importance of this channel is minor in any case. Eliminating differences in  $\alpha$  only affects the difference between sectoral factor income shares, but not their changes.

Differences in  $\sigma$  and in growth rates clearly have effects of a different calibre. Line 5 of the table reports results for an economy where the factor imbalance of productivity growth is the same in both sectors. It is clear from equation (2) that this imbalance is a key contributor to changes in factor income shares. In the benchmark calibration, the

Table 4: Counterfactuals

		change in					
		manufacturing labor income share	services labor income share	aggregate labor income share	manufacturing employment share	manufacturing share of capital	manufacturing share of value added
1.	data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
2.	benchmark	-6.0	-3.1	-4.4	-23.9	-20.7	-22.9
3.	$s = 0$ (homothetic preferences)	-6.1	-3.2	-4.5	-23.3	-20.2	-22.3
4.	common $\bar{\theta}$ (weight on capital in production)	-6.1	-3.1	-6.0	-25.4	-22.7	-24.5
5.	common factor imbalance of productivity growth	-4.3	-10.8	-9.2	-17.0	-22.0	-19.3
6.	common growth rates ( $g(A_i)$ and $g(\tilde{B}_i)$ )	-16.0	-31.9	-22.9	10.5	-4.8	2.2
7.	common $\sigma$ (capital/labor elasticity of substitution)	-8.9	-2.5	-4.1	-22.9	-16.3	-20.8

Notes: All changes are in percentage points. Except for the parameters that are equated, all parameters as in the benchmark. Data source for line 1: as in Figure 2. Line 4:  $\bar{\theta}_i$  in both sectors is set to the average of  $\bar{\theta}_m$  and  $\bar{\theta}_s$  in Table 3, keeping  $A_{m0}$  and  $A_{s0}$  unchanged. Line 5:  $g(\tilde{B}_i)$  is set to -0.0817, its average value in the calibration, for both sectors. Then, the levels of neutral productivity growth are shifted to ensure that the growth rate of output is the same as in the benchmark economy. This implies  $g(A_m) = 0.0707$ ,  $g(\tilde{B}_m) = -0.0817$ ,  $g(A_s) = 0.0156$  and  $g(\tilde{B}_s) = -0.0817$ . Line 6:  $g(A_m) = g(A_s) = 0.0385$  and  $g(\tilde{B}_m) = g(\tilde{B}_s) = -0.082$ . These values imply that the growth rate of output is the same as in the benchmark economy, and the average factor imbalance of productivity growth is preserved. Line 7:  $\sigma_m = \sigma_s = 0.673$ . This is the average of  $\sigma_m$  and  $\sigma_s$ .



imbalance in manufacturing exceeds that in services by slightly more than 2 percentage points. Equating it in the two sectors, i.e. reducing it in manufacturing and raising it in services, then implies that the change in the capital income share in manufacturing is reduced, while that in services is amplified. This occurs to a point where the change in income shares in services exceeds that in manufacturing, exactly as predicted by equation (2) for the case of equal factor imbalance of productivity growth and different  $\sigma_i$ . Given the increasing importance of services, this change also implies a larger decline in the aggregate labor income share.

Structural change is also affected by the imbalance of productivity growth. In general, two forces determine the evolution of manufacturing capital intensity. First, in the benchmark economy, effective labor becomes relatively more abundant, and thus relatively cheaper, over time. Since the elasticity of substitution in manufacturing exceeds that in services, producers in manufacturing are better placed to take advantage of this relative price change. As a consequence, labor intensity in manufacturing tends to increase. Second, for a given elasticity of substitution, a reduction in  $g(\tilde{B}_i)$  in any sector requires increasing the capital intensity of production in that sector, given that capital and labor are gross complements. Hence, the stronger imbalance of productivity growth in manufacturing tends to increase capital intensity in that sector. This effect counteracts the first force. In the benchmark economy, the interaction of both forces leads to a minor increase in manufacturing capital intensity over our sample period. Equating the factor imbalance of productivity growth across sectors in the counterfactual implies disabling the second channel. As a result, manufacturing becomes less capital-intensive in this counterfactual.

Line 6 reports results for a counterfactual economy where all productivity growth rates are equated across sectors. The effects on changes in the labor income share are qualitatively similar to those of changing only  $g(\tilde{B}_i)$ , but they are magnified. The equalization of growth rates across sectors, however, leads to a reduction in capital accumulation (which in the benchmark is boosted by strong productivity growth in manufacturing). With  $\sigma < 1$  in both sectors, this raises the capital income share, and more so in the sector that is further away from Cobb-Douglas (services). Eliminating growth rate differences also essentially stops structural change. The slowdown of capital accumulation implies that compared to the previous line, labor becomes more abundant in the economy. As a result, the more flexible sector, manufacturing, becomes more labor-intensive. Since overall there is hardly any structural change in this case, this means that the share of labor employed in manufacturing increases, while its share of capital declines.

The difference in  $\sigma_i$  also has a substantial effect both on changes in factor income shares and on the shape of structural change. In the benchmark, services are further away from Cobb-Douglas, implying that for a given factor imbalance of productivity growth, the labor income share in services changes more. (Recall that this is why the benchmark calibration features a larger  $g(\tilde{B}_i)$  in manufacturing.) Eliminating the difference in  $\sigma_i$  implies making the manufacturing sector less flexible, so that its labor income share declines even more. Services, in contrast, become more flexible, leading to a smaller decline in the labor income share in that sector.

Finally, while differences in the elasticity of substitution are not the main driver of structural change, they are key for getting the shape of structural change right. Without  $\sigma_i$  differences, manufacturing, with its larger factor imbalance of production growth and its larger  $g(A_i)$ , shifts towards more capital-intensive production. Combined with an overall movement out of manufacturing, this implies that manufacturing sheds capital substantially more slowly than it sheds labor – differently from the data.

Observed movements in labor income shares and in sectoral allocations thus depend crucially on growth rate differences across sectors, and on differences in substitution possibilities. First, matching changing sectoral factor income shares in a model with competitive labor markets requires substitution elasticities below unity. Second, matching different rates of change in these shares across sectors requires either differences in the factor imbalance of productivity growth or differences in substitution elasticities. Finally, lines 5 and 7 of Table 4 indicate that just one difference is not enough: matching the faster decline in the labor income share in manufacturing requires either a common factor imbalance and a higher  $\sigma_i$  in services (supposing that  $\sigma_i < 1$  in both sectors), or a common  $\sigma_i$  and a larger factor imbalance in manufacturing. In both cases, the shape of structural change would be different from the data: given that overall, effective labor becomes more abundant relative to effective capital, the relative capital-intensity of manufacturing would increase in both cases.<sup>36</sup> This contrasts with the data, where it is virtually constant. Hence, the joint analysis of the evolution of sectoral factor income shares and structural change leads us to the conclusion that sectoral differences both in factor substitutability and in factor-specific rates of technical change are required to match observed trends.

Returning to the trends in the data, our results suggest that declines in sectoral labor income shares have been driven by strongly capital-biased technical change. This is akin

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<sup>36</sup>In the first case, services – here hypothetically the more flexible sector – would increase intensity of use of the factor that becomes more abundant: labor. In the second case, as labor becomes more abundant in manufacturing more quickly than in services, manufacturing would increase its capital intensity.

to effective labor becoming more abundant relative to effective capital in a world where the two inputs are gross complements. This feeds through from sectors to the aggregate labor income share. The decline in the manufacturing labor income share is particularly severe because the factor imbalance in productivity growth has been stronger in that sector.

## 5 Conclusion

In this paper, we trace the decline in the aggregate labor income share, which has occurred in several developed countries, to developments at the sectoral level. In line with this, we establish a connection between this decline and another important development that has been taking place over the last half century: large reallocations of resources and production from manufacturing to services.

We show that to account for both developments, a simple dynamic two-sector model needs to feature cross-sectoral differences both in capital-labor substitutability and in the rate and bias of technical change. Overall, the key driver of the decline in the labor income share is capital-biased technical change, combined with the complementarity of capital and labor in production. Intuitively, capital-biased technical change raises the marginal product of capital relative to that of labor, dominating the opposite effect of capital accumulation on relative income shares. Put differently, with complementarity, the increase in the relative abundance of effective labor implied by capital-biased technical change reduces the labor income share.

This mechanism also affects structural change, in an intuitive way. (See Alvarez-Cuadrado et al. (2017) for more detail.) As the relative abundance of inputs of production changes with the process of economic growth, so does their relative price. In the presence of sectoral differences in the elasticity of substitution, the more flexible sector will increase its use of the relatively cheaper factor. While our quantitative exercise clearly points to differences in productivity growth rates across sectors as the main driver of structural change, this new mechanism is required to match observed patterns of structural change in terms of capital and labor.

In this context, it is natural to wonder what kept the U.S. labor income share roughly constant for most of the last century. One possibility is that, as a result of structural change, sectoral changes canceled out in the aggregate. The careful estimates of sectoral labor income shares constructed by Valentinyi and Herrendorf (2008) show that in 1997, the labor income share in non-agriculture was 50% larger than that in agriculture, a difference of 21 percentage points. Caselli and Coleman II (2001) document a decrease in U.S. agricultural

employment of 40 percentage points between 1880 and 1960. Over this period, the fraction of value added produced in agriculture declined by 21 percentage points. Combining these pieces of evidence, structural change out of agriculture by itself would generate an increase in the aggregate labor income share of the order of 4.5 percentage points. This development would have counteracted a decline in the labor income share driven by manufacturing. In view of these calculations, it is possible that at the aggregate level, changes in the labor income shares in manufacturing and agriculture cancelled each other. This would be in line with Keynes's (1939) observation (cited in Elsby et al. 2013, p. 13) that the "remarkable constancy" of the aggregate labor share is "a bit of a miracle".

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# Appendix

## A Data

### A.1 Computation of labor income shares

This section closely follows Valentinyi and Herrendorf (2008). Let the number of commodities be  $M$  and the number of industries be  $I$ . Index commodities by  $m$  and industries by  $i$ . From the 35-sector KLEM data, we compute the following objects:

- The “use matrix”  $\mathbf{B}$ : this is an  $M \times I$  matrix with representative element  $(m, i)$  that states how much output of commodity  $m$  is required to produce 1\$ of output of industry  $i$ .
- The “make matrix”  $\mathbf{W}$ : this is an  $I \times M$  matrix with representative element  $(i, m)$  that states for industry  $i$  which share of commodity  $m$  it produces.
- The final expenditure vector  $\mathbf{y}$ : this length- $I$  column vector states the amount of output of each industry  $i$  that serves as final expenditure.

In our setting, we have no commodity level information and have to assume that each industry produces a single commodity. Therefore,  $M = I$ .

The final expenditure vector  $\mathbf{y}$  can be computed by subtracting each industry’s output that is used as an intermediate input in another industry from the total value of its output. The use matrix  $\mathbf{B}$  can be computed by dividing the value of intermediate inputs from industry  $M$  used in industry  $I$  by the value of gross output in industry  $I$ . When each industry produces a single, unique commodity, the make matrix  $\mathbf{W}$  simply is the  $I \times I$  identity matrix.

Let the column vector of sectoral shares of labor income in the value of gross output be  $\alpha_l$  and the column vector of the sectoral shares of capital income in the value of gross output be  $\alpha_k$ . Let the  $I \times 1$  sector identification vector with elements  $\mathbb{1}_{i=j}$  for a sector  $j$  be  $\mathbf{1}_j$  and let  $\mathbf{y}_j \equiv \mathbf{y}'\mathbf{1}_j$ . Following Valentinyi and Herrendorf (2008), we then obtain the share of labor income in value added in sector  $j$  as

$$\frac{\alpha_l' \mathbf{W} (\mathbf{I} - \mathbf{B} \mathbf{W})^{-1} \mathbf{y}_j}{(\alpha_l' + \alpha_k') (\mathbf{I} - \mathbf{B} \mathbf{W})^{-1} \mathbf{y}_j}, \quad (20)$$

### A.2 Sector classification.

**U.S. data:** Using data from Jorgenson’s (2007) 35-sector data base, we construct sectors as follows:

**Manufacturing:** Sectors 7-27: Food and kindred products, Tobacco, Textile mill products, Apparel, Lumber and wood, Furniture and fixtures, Paper and allied, Printing, publishing and allied, Chemicals, Petroleum and coal products, Rubber and misc plastics, Leather, Stone, clay, glass, Primary metal, Fabricated metal, Non-electrical machinery, Electrical machinery, Motor vehicles, Transportation equipment & ordnance, Instruments, Misc. manufacturing

**Services:** Transportation, Communications, Electric utilities, Gas utilities, Trade, Finance Insurance and Real Estate, Other Services

Results from Section 2 are not sensitive to excluding Utilities from Services or to including Mining and Construction with Manufacturing, or Government in Services.

**Cross-country data:** Using EU-KLEMS data from <http://www.euklems.net>, we define manufacturing analogously to the U.S. data. Again, as there, we include utilities (sector E), wholesale and retail trade (G), hotels and restaurants (H), transport, storage and communication (I), financial intermediation (J), real estate etc. (K) in services. We exclude public administration and defence (L), education (M), health and social work (N), other community services (O), private households (P).

### **A.3 The measurement of industry factor income shares.**

Jorgenson et al. (1987) contains a detailed description of measurement of industry-level factor income shares. The following are some key features. Labor compensation is measured as wage and salary income plus supplements, including employers' contribution to Social Security and unemployment compensation contributions by employers. Annual measures are computed using time actually worked. Earnings of the self-employed are split between capital and labor compensation assuming an after-tax rate of return that matches that of corporate businesses.

## B Details on Section 3

### B.1 Details on the model solution

Time is discrete.<sup>37</sup> We model a closed economy populated by a representative households with preferences given by,

$$v = \ln(u(c_m, c_s)) \quad \text{where} \quad u(c_m, c_s) = \left( \gamma c_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_s + s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (21)$$

where the variables and parameters are described in the body of the paper and per period utility is discounted at a rate  $\beta$ . Sectoral outputs are produced according to the general CES technologies

$$Y_m = A_m \left[ \alpha_m (\tilde{B}_m K_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m) L_m^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} \quad (22)$$

$$Y_s = A_s \left[ \alpha_s (\tilde{B}_s K_s)^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s) L_s^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}}. \quad (23)$$

Factors are fully utilized. Normalizing the labor endowment to one, this implies

$$\frac{L_m}{L} + \frac{L_s}{L} \equiv l_m + l_s = 1 \quad (24)$$

$$l_m k_m + l_s k_s = l_m k_m + (1-l_m) k_s = k, \quad \text{where } k_i \equiv \frac{K_i}{L_i}. \quad (25)$$

We choose manufactures to be the numeraire and denote the price of services by  $p_s$ . Since markets are competitive, production efficiency requires equating marginal revenue products across sectors, so

$$\alpha_m \left( A_m \tilde{B}_m \right)^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{K_m} \right)^{\frac{1}{\sigma_m}} = R = p_s \alpha_s \left( A_s \tilde{B}_s \right)^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{K_s} \right)^{\frac{1}{\sigma_s}} \quad (26)$$

$$(1-\alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{L_m} \right)^{\frac{1}{\sigma_m}} = w = p_s (1-\alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{L_s} \right)^{\frac{1}{\sigma_s}}. \quad (27)$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left( \frac{1-\alpha_m}{1-\alpha_s} \frac{\alpha_s}{\alpha_m} \tilde{B}_m^{\frac{1-\sigma_m}{\sigma_m}} \right)^{\sigma_s} \tilde{B}_s^{\sigma_s-1} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}, \quad (28)$$

where  $\varrho \equiv \left( \frac{1-\alpha_m}{1-\alpha_s} \frac{\alpha_s}{\alpha_m} \tilde{B}_m^{\frac{1-\sigma_m}{\sigma_m}} \right)^{\sigma_s} \tilde{B}_s^{\sigma_s-1}$ . Then, using (8), the economy-wide capital-labor ratio can be written as

$$k = l_m k_m + l_s k_s = l_m k_m + (1-l_m) \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \quad (29)$$

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<sup>37</sup>For ease of notation, we limit the use of time subscripts to expressions that include variables dated at two points in time.

Using this notation, the relative price is given by

$$p_s = \frac{\alpha_m A_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}}}{\alpha_s A_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}}} \frac{\left[ \alpha_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) k_m^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}{\left[ \alpha_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (\rho k_m^{\frac{\sigma_s}{\sigma_m}})^{\frac{1-\sigma_s}{\sigma_s}} \right]^{\frac{1}{\sigma_s-1}}}. \quad (30)$$

On the household side, the representative agent equates the marginal rate of substitution between the two goods to their relative price, so

$$\frac{v_s}{v_m} = \frac{1 - \gamma}{\gamma} \left( \frac{c_s + s}{c_m} \right)^{-\frac{1}{\varepsilon}} = p_s. \quad (31)$$

since  $v_s = \frac{u_s}{u} = u^{\frac{1-\varepsilon}{\varepsilon}} (1 - \gamma) (c_s + s)^{-\frac{1}{\varepsilon}}$ . Define the following variables.

$$x_s \equiv \frac{p_s c_s}{c_m} \quad \text{and} \quad x_m \equiv \frac{c_m}{c_m} = 1. \quad (32)$$

Then, total consumption expenditure  $c = p_s c_s + c_m$  relative to consumption expenditure on manufactures is given by

$$X = x_s + x_m = \frac{p_s c_s + c_m}{c_m} \equiv \frac{c}{c_m}. \quad (33)$$

Now we are in a position to determine the static allocation of resources across sectors. Combining (30), (32), and (33) we can express consumption of services as

$$c_s = \frac{x_s c_m}{p_s} = x_s \frac{c}{X} \frac{\alpha_s A_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}}}{\alpha_m A_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}}} \frac{\left[ \alpha_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (\rho k_m^{\frac{\sigma_s}{\sigma_m}})^{\frac{1-\sigma_s}{\sigma_s}} \right]^{\frac{1}{\sigma_s-1}}}{\left[ \alpha_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) k_m^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}. \quad (34)$$

Since service output is fully consumed,  $c_s$  is also given by

$$c_s = y_s = A_s \left[ \alpha_s \left( \tilde{B}_s \rho k_m^{\frac{\sigma_s}{\sigma_m}} \right)^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) \right]^{\frac{\sigma_s}{\sigma_s-1}} l_s. \quad (35)$$

Combining the previous two expressions, we can solve for  $l_m$  as a function of current values of the endogenous variables,  $k_m$  and  $c_m$ , given the exogenous levels of labor- and capital-augmenting productivity.

$$l_m = 1 - \frac{x_s}{X} c \frac{\alpha_s}{\alpha_m} \frac{\tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}}}{A_m \tilde{B}_m^{\frac{\sigma_m-1}{\sigma_m}}} \frac{\left[ \alpha_m \left( \tilde{B}_m k_m \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) \right]^{\frac{1}{1-\sigma_m}}}{\left[ \alpha_s \tilde{B}_s^{\frac{\sigma_s-1}{\sigma_s}} \rho k_m^{\frac{\sigma_s}{\sigma_m}} + (1 - \alpha_s) \rho^{\frac{1}{\sigma_s}} \right]} \quad (36)$$

This labor allocation together with the full employment condition for capital given by (29) determines the current sectoral allocation of resources.

Now we turn to the dynamic evolution of the economy. Since capital accumulates out of manufacturing output, the per capita capital stock evolves according to

$$(1+n)k_{t+1} = y_{m,t}f_m(k_{m,t})l_{m,t} - c_{m,t} + (1-\delta)k_t, \quad (37)$$

where

$$y_{m,t} \equiv f_m(k_{m,t})l_{m,t} = A_{m,t} \left[ \alpha_m \left( \tilde{B}_{m,t} k_{m,t} \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m) \right]^{\frac{\sigma_m}{\sigma_m-1}} l_{m,t}, \quad (38)$$

is manufacturing output per capita.

Combining (37) with (29), the law of motion of  $k_{m,t}$  is given by

$$k_{m,t+1} = \frac{1}{(1+n)l_{m,t+1}} \left[ y_{m,t} - \frac{c_t}{X_t} + (1-\delta) \left( l_{m,t}k_{m,t} + (1-l_{m,t})\varrho_t k_{m,t}^{\frac{\sigma_s}{\sigma_m}} \right) - (1+n)(1-l_{m,t+1})\varrho_{t+1}k_{m,t+1}^{\frac{\sigma_s}{\sigma_m}} \right]. \quad (39)$$

The Euler equation for this problem is given by

$$\nu_m(c_{m,t+1}, c_{s,t+1}) \beta [f'_m(k_{m,t+1}) + 1 - \delta] = (1+n)\nu_m(c_{m,t}, c_{s,t}), \quad (40)$$

Since preferences are homogeneous of degree 1,

$$u = u_m c_m + u_s (c_s + s) = u_m c_m + p_s u_m (c_s + s) = (c_m + p_s (c_s + s)) u_m = (c + p_s s) u_m, \quad (41)$$

where the second equality uses  $\nu_m = u_m/u$  and (31) and the last one uses (33). As a result,

$$\nu_m(c_{m,t}, c_{s,t}) = \frac{1}{c_t + p_{s,t}s},$$

and the Euler equation becomes

$$c_{t+1} = \frac{\beta}{1+n} [f'_m(k_{m,t+1}) + 1 - \delta] (c_t + p_{s,t}s) - p_{s,t+1}s. \quad (42)$$

Equations (39) and (42) form a system of difference equations in  $k_{m,t}$  and  $c_t$  that together with (30), (33), (36), (38) and the fact that  $x_{s,t} = \left( \left( \frac{1-\gamma}{\gamma} \right)^\varepsilon p_{s,t}^{1-\varepsilon} - \frac{p_{s,t}s}{c_t} \right) / \left( 1 + \frac{p_{s,t}s}{c_t} \right)$  fully describe the dynamics of the economy.

## B.2 The Aggregate Elasticity of Substitution

This proof follows Jones (1965) and Miyagiwa and Papageorgiou (2007). Time subscripts are omitted throughout. The dual relationship between sectoral prices and input prices and factor endowments and sectoral outputs are given by

$$C_s(w, R) = p_s \quad (43)$$

$$C_m(w, R) = p_m \quad (44)$$

$$Y_s C_{sw} + Y_m C_{mw} = L \quad (45)$$

$$Y_s C_{sR} + Y_m C_{mR} = K \quad (46)$$

where  $C_i(w, R)$  is the minimum unit cost function for sector  $i = s, m$ . It is linearly homogeneous.  $C_{ij}$  are its partial derivatives with respect to each factor price  $j = w, R$ . By Shepard's lemma,

$$C_{iw} = L_i/Y_i \quad (47)$$

$$C_{iR} = K_i/Y_i \quad (48)$$

are the unit factor demands.

Differentiating the previous expressions we reach the following relationships, where for compactness we denote a change as  $dx = x_{t+1} - x_t$  and percentage change as  $\hat{x} \equiv \frac{dx}{x}$ .

$$(1 - \epsilon_s) \hat{w} + \epsilon_s \hat{R} = \hat{p}_s \quad (49)$$

$$(1 - \epsilon_m) \hat{w} + \epsilon_m \hat{R} = \hat{p}_m \quad (50)$$

$$l_s (\hat{Y}_s + \hat{C}_{sw}) + (1 - l_s) (\hat{Y}_m + \hat{C}_{mw}) = \hat{L} \quad (51)$$

$$\kappa_s (\hat{Y}_s + \hat{C}_{sR}) + (1 - \kappa_s) (\hat{Y}_m + \hat{C}_{mR}) = \hat{K}, \quad (52)$$

where we use the fact that sectoral production functions are homogeneous of degree one.  $\epsilon_i$  denotes the capital income share in sector  $i = s, m$ , and  $l_s \equiv L_s/L$  and  $\kappa_s \equiv K_s/K$  are the fractions of labor and capital used in the service sector, respectively.

Subtracting (49) from (50),

$$(\epsilon_m - \epsilon_s) (\hat{w} - \hat{R}) = \hat{p}_s - \hat{p}_m. \quad (53)$$

Rewriting the sector-specific elasticity of substitution  $\sigma_i$  as  $\frac{C_i C_{iwR}}{C_{iw} C_{iR}}$  and expressing the factor income shares as  $\epsilon_i = \frac{r C_{iR}}{C_i}$  and  $1 - \epsilon_i = \frac{w C_{iw}}{C_i}$ , we reach the following rates of change of partial derivatives of the unit cost functions.<sup>38</sup>

$$\begin{aligned} \hat{C}_{iw} &= \frac{C_{iww} dw + C_{iwr} dR}{C_{iw}} = \frac{-C_{iwr} \frac{R}{w} dw + C_{iwr} dR}{C_{iw}} = \\ &= -\frac{(C_{iwr} R \hat{w} - C_{iwr} dR)}{C_{iw}} = -\frac{C_{iwr} R}{C_{iw}} (\hat{w} - \hat{R}) = -\frac{C_i C_{iwr}}{C_{iw} C_{iR}} \frac{R C_{iR}}{C_i} (\hat{w} - \hat{R}), \end{aligned}$$

where the second equality uses the fact that  $C_{iw}(w, R)$  is homogeneous of degree 0. As a result,

$$\hat{C}_{iw} = -\sigma_i \epsilon_i (\hat{w} - \hat{R}) \quad (54)$$

<sup>38</sup>The standard definition of the elasticity of substitution between capital and labor is given by  $\sigma_i \equiv \frac{d \ln(K_i/L_i)}{d \ln(w/R)} = \frac{d \ln(C_{iR}/C_{iw})}{d \ln(w/R)}$ , where the last expression uses (47) and (48). The numerator can be written as  $d \ln(C_{iR}/C_{iw}) = \frac{(C_{iRw} C_{iw} - C_{iww} C_{iR}) dw + (C_{iRR} C_{iw} - C_{iwr} C_{iR}) dR}{C_{iR} C_{iw}}$ . Since  $C_{ij}$  is homogeneous of degree zero,  $C_{iww} = -C_{iwr} \frac{R}{w}$  and  $C_{iRR} = -C_{iRw} \frac{w}{R}$ . Inserting these in the previous expression, we reach  $d \ln(C_{iR}/C_{iw}) = \frac{C_{iRw}}{C_{iR} C_{iw}} d \ln(w/R)$ , which leads to the definition in the text.

$$\hat{C}_{iR} = \sigma_i (1 - \epsilon_i) (\hat{w} - \hat{R}) \quad (55)$$

Replacing (54) and (55) in (51) and (52) and subtracting them we reach,

$$(l_s - \kappa_s) (\hat{Y}_s - \hat{Y}_m) = (\hat{L} - \hat{K}) + \Theta (\hat{w} - \hat{R}) \quad (56)$$

where  $\Theta \equiv l_s \sigma_s \epsilon_s + (1 - l_s) \sigma_m \epsilon_m + \kappa_s \sigma_s (1 - \epsilon_s) + (1 - \kappa_s) \sigma_m (1 - \epsilon_m)$ .

Then we combine (31) and (53) to reach

$$\frac{dc_s}{c_s + s} - \hat{c}_m = -\varepsilon (\epsilon_m - \epsilon_s) (\hat{w} - \hat{R}) \quad (57)$$

Since the aggregate elasticity of substitution is defined as  $\sigma \equiv -\frac{(\hat{L}-\hat{K})}{(\hat{w}-\hat{R})}$  we combine the fact that services are fully consumed, so  $Y_s = Lc_s$ , (37), (56) and (57) to reach,

$$\sigma = \gamma_0 \varepsilon + \gamma_1 \sigma_s + \gamma_2 \sigma_m + \frac{(l_s - \kappa_s)}{(\hat{w} - \hat{R})} \left( \hat{Y}_m - n - \hat{c}_m \left( 1 + \frac{s}{c_s} \right) \right) \quad (58)$$

where,

$$\begin{aligned} \gamma_0 &\equiv (\epsilon_m - \epsilon_s) (l_s - \kappa_s) \left( 1 + \frac{s}{c_s} \right) \\ \gamma_1 &\equiv l_s \epsilon_s + \kappa_s (1 - \epsilon_s) \\ \gamma_2 &\equiv (1 - l_s) \epsilon_m + (1 - \kappa_s) (1 - \epsilon_m) \end{aligned}$$

## C Robustness: Quantitative results for alternative settings

### C.1 Quantitative results with $\varepsilon = 0.5$

We also compute results for an economy where preferences over manufacturing and services consumption are not Leontief. Just as Buera and Kaboski (2009), we consider a value of  $\varepsilon$  of 0.5. While far away from Leontief, this still implies that manufacturing and services output clearly are gross complements in consumption. We recalibrate the model in this setting, and conduct the same decomposition as above.

Table 5 shows calibration results. The calibration for higher  $\varepsilon$  clearly fits less well. While the model replicates changes in the factor income shares and structural change well, it proved impossible to get levels of factor income shares and the initial labor allocation to fit as closely as in our benchmark calibration. In particular, the average labor income share in services does not fit well. This also affects the fit of the aggregate labor income share. Note that changes in both the labor income share and in allocations do fit rather well, though.

Table 5: Calibration for  $\varepsilon = 0.5$ .*Panel A: Targets and model moments*

	Data		Model	
	Manufacturing	Services	Manufacturing	Services
<i>calibration targets:</i>				
$\overline{LIS}_i$ (%)	66.7	63.8	65.7	71.9
$g(KIS_i/LIS_i)$ (% p.a.)	0.6%	0.3%	0.60%	0.31%
$d(L_m/L)$	-22.4		-22.6	
$d(K_m/K)$	-21.3		-20.9	
$L_m/L$	34.2		36.5	
$g(Y/L)$ (% p.a.)	1.8%		1.78%	
<i>not targeted:</i>				
$\overline{LIS}$ (aggregate, %)	64.8		69.5	
$g(KIS/LIS)$ (agg., % p.a.)	0.5		0.3	
$\overline{K_m/K}$	33.2		44.1	
$\overline{Y_m/Y}$	35.1		38.6	
$d(Y_m/Y)$	-24.5		-21.9	

*Panel B: Parameters*

<i>sector-specific:</i>	Manufacturing	Services	<i>general:</i>	
$g(A_i)$ (% p.a.)	9.8	0.8	$\delta$	0.05
$g(B_i)$ (% p.a.)	-2.6	-6.9	$\beta$	0.94
$\sigma_i$	0.875	0.534		
$A_{i1}$	0.641	1.143		
$\bar{\theta}_i$ (normalization)	0.333	0.362		

Note: The data period used is 1960 to 2005. Averages are geometric means over this period. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.

Parameters are overall similar; the manufacturing sector is more flexible, and growth rate patterns are as above, driven again by the same data patterns.

Broadly speaking, decomposition results are similar to those in the Leontief case. Non-homotheticity of preferences hardly affects changes in labor income shares or structural change. Common  $\alpha$  here has some effect, though this is also due to the fact that in this calibration, the average labor income share in services is off by 8 percentage points. Differences



Table 6: Counterfactuals ( $\varepsilon = 0.5$ )

	dLIS <sub>m</sub>	dLIS <sub>s</sub>	dLIS (aggregate)	d(L <sub>m</sub> /L)	d(K <sub>m</sub> /K)	d(Y <sub>m</sub> /Y)
data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
best fit	-6.1	-2.8	-2.4	-22.6	-20.9	-21.9
$s = 0$	-6.1	-2.9	-2.7	-21.6	-19.6	-20.8
common $\bar{\theta}$	-5.0	0.3	-0.4	-24.5	-20.6	-23.4
common $g(A)$	-11.8	-44.1	-25.6	14.4	-18.9	-0.6
common $\sigma$	-15.2	-2.6	-6.0	-17.5	-6.0	-13.8

Notes: LIS stands for “labor income share” and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Except for the parameters that are equated, all parameters as in Table 5. Line 4:  $\bar{\theta}_i$  in both sectors is set to the average of  $\bar{\theta}_m$  and  $\bar{\theta}_s$  in Table 5, keeping  $A_{m1}$  and  $A_{s1}$  unchanged. Line 5:  $g(A_m) = g(A_s) = 0.0462$  and  $g(B_m) = g(B_s) = -0.0543$ . This value implies that the growth rate of  $y$  is the same as in the benchmark economy, and the average distance between  $g(A_i)$  and  $g(B_i)$  is preserved. Line 6:  $\sigma_m = \sigma_s = 0.705$ . This is the average of  $\sigma_m$  and  $\sigma_s$ .

in  $\sigma$  and in growth rates again prove to be the most important determinants of changes in labor income shares and structural change. Eliminating differences in growth rates essentially eliminates structural change in terms of value added; the value added share of manufacturing falls by only half a percentage point. There is still substantial factor reallocation because of  $\sigma$  differences, though: Since overall, growth rates are such that efficiency units of labor become more abundant relative to capital, the more flexible sector (manufacturing) substitutes towards that more abundant input. Without structural change on average, this implies that labor flows into manufacturing, while the fraction of capital used in the sector declines.

Eliminating differences in  $\sigma$  in this setting affects not only the shape but also the amount of structural change. As above, a larger difference between  $g(A)$  and  $g(B)$  in manufacturing implies that if  $\sigma$  is equal, labor moves towards services more quickly than capital does. With structural change taking place at the same time, this results in a decline of the fraction of capital used in manufacturing that is slower than the decline of the fraction of labor employed in manufacturing. The changes in sectoral labor income shares are in line with this. Differences in growth rates tend to lead to a faster decline of the labor income share in manufacturing. The higher substitution elasticity in manufacturing counteracts this; eliminating the  $\sigma$  difference then makes the labor income share in manufacturing decline much faster than that in services. The largest difference between this case and the benchmark is that equating  $\sigma$  affects not only the shape of structural change, but also how much structural change takes place overall: the decline in the share of value added produced in the manufacturing sector drops from 22.9 percentage points in the benchmark calibration to 13.8

percentage points here.

Overall, these results are in line with our benchmark results above. We can thus conclude that it is the combination of differences in  $\sigma$  and in growth rates across sectors that are key for understanding the evolution of factor income shares and structural change from manufacturing to services.

## C.2 Quantitative results with Cobb-Douglas production functions

Table 7: Calibration: Parameters (Cobb-Douglas).

<i>sector-specific:</i>	Manufacturing	Services
growth of $A_i$ (neutral technology, % p.a.)	6.2	-1.7
$\alpha$ (elasticity of output with respect to capital)	0.333	0.362

Table 8: Calibration: Targets and model moments (Cobb-Douglas).

	Data			Model		
	Manufacturing	Services	Aggregate	Manufacturing	Services	Aggregate
<i>calibration targets:</i>						
average sectoral LIS (%)	66.7	63.8		66.7	63.8	
growth in sectoral KIS/LIS (% p.a.)	0	0		0	0	
change in manufacturing employment share	-22.4			-22.4		
change in manufacturing share of capital	-21.3			-21.7		
growth in output per worker (% p.a.)			1.8			1.8
<i>not targeted:</i>						
average aggregate LIS (%)			64.8			64.9
growth in aggregate KIS/LIS (% p.a.)			0.5			0.06
average manufacturing share of capital	33.2			35.2		
average manufacturing value added share	35.1			37.0		
change in manufacturing value added share	-24.5			-22.2		

Note: The data period used is 1960 to 2005. LIS (KIS) stands for the labor (capital) income share. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.